(New) Gauge/Gravity duality with Anisotropies

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Outline

Introduction to AdS/CFT– The Road to 'Reality'

- The initial AdS/CFT correspondence is the harmonic oscillator of the gauge/gravity dualities: $\mathcal{N} = 4$ sYM on flat space $\Leftrightarrow AdS_5 \times S^5$.
- Since the discovery of the initial correspondence, there is an extensive research aiming to construct gauge/gravity dualities that can be thought as toy models to describe realistic systems and theories. Hope for universal behaviors!

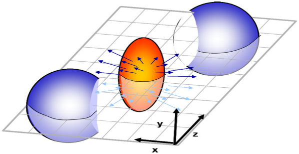
  Have been constructed Gauge/Gravity Dualities with: Less Supersymmetry; Broken conformal symmetry, confinement; fundamental matter(probe and backreacting Dq branes); etc.

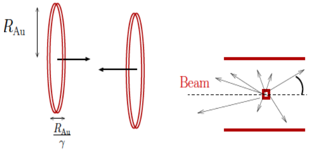
✓ We include Anisotropy → New Gauge/Gravity duality.

Why? An Argument for 'applications'

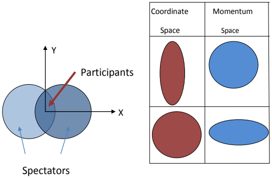
- The existence of strongly coupled anisotropic systems.
  Examples: The expansion of the plasma along the longitudinal beam axis at the earliest times after the collision results to momentum anisotropic plasmas.
  Strong Magnetic Fields in strongly coupled theories.
  Anisotropic low dimensional materials in condensed matter.

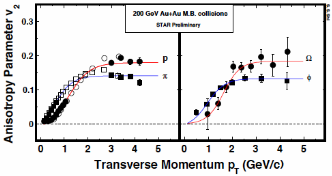
Example:Elliptic flow





Pressure gradients for non-central collisions along the short axis of the elliptic flow are higher than the long axis. Therefore the expansion along the short axis is more rapid leading to anisotropic momentum distribution.





The nuclear medium expands preferably along he short axis of the ellipse. The elliptic flow parameter

$$v_2 = \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$

can be measured experimentally through the particle distributions.

Why? More:

- Weakly coupled vs strongly coupled anisotropic theories.
- Consistent top-down models. Properties of the supergravity solutions, that are dual to the anisotropic theories.
- AdS in UV to Lifhitz-like in IR **flows**:
  ★ Why there is a fixed scaling parameter z for such solutions?
  ★ Other systems that have fixed scaling IR solution (e.g. in Heavy quark density). Why?
  (Kumar 2012; Faedo, Kundu, Mateos, Tarrio 2014)
  ★ Hyperscaling violation IR solutions?
- Ⓢtriking Features! Several Universality Relations predicted for the isotropic theories are violated!
  ★ Shear viscosity over entropy density ratio takes parametrically low values!
  ★ Langevin coefficients inequality between the transverse and parallel to the heavy quark motion component, gets inverted!
  ★ Implications to QGP hydrodynamic simulations.

Reminding Slide: 1

The Lifshitz-like space, where some spatial dimensions scale in a different way with the rest:

$$ds^2 = r^{2z}\left(-dt^2 + dy_i^2\right) + r^2 dx_j^2 + \frac{dr^2}{r^2} ,$$

where z is a scaling parameter and $i + j = 1, ..., d$. The metric is invariant under

$$t \to \lambda^z t, \qquad y \to \lambda^z y, \qquad x \to \lambda x, \qquad r \to \frac{r}{\lambda} .$$

Equivalently the coordinate transformation $r \to r^{1/z}$, gives

$$ds^2 = r^2\left(-dt^2 + dy_i^2\right) + r^{2/z} dx_j^2 + \frac{dr^2}{r^2} ,$$

where the x_j directions are the anisotropic ones.

Reminding Slide: 2

The anisotropic hyperscaling violation metric

$$ds^2 = r^{-\frac{2\theta}{d}} \left(-r^{2z} \left(dt^2 + dy_i^2 \right) + r^2 dx_j^2 + \frac{dr^2}{r^2} \right) ,$$

which exhibits a critical exponent z and a hyperscaling violation exponent θ. The metric is not scale invariant as

$$t \to \lambda^z t, \qquad y \to \lambda^z y, \qquad x \to \lambda x, \qquad r \to \frac{r}{\lambda} , \qquad ds \to \lambda^{\frac{\theta}{d}} ds \ .$$

The coordinate transformation $r \to r^{1/z}$ bring the metric in the form

$$ds^2 = r^{-\frac{2\theta}{dz}} \left(-r^2 \left(dt^2 + dy_i^2 \right) + r^{\frac{2}{z}} dx_i^2 + \frac{dr^2}{r^2} \right) .$$

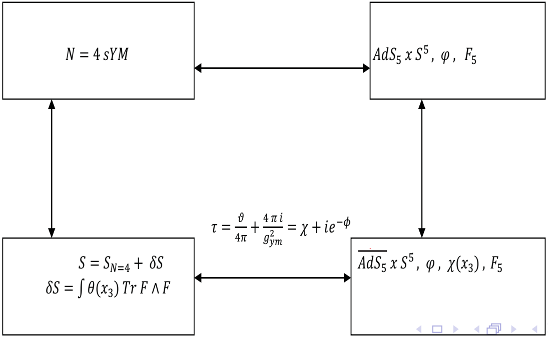
which reveals clearly the anisotropic directions.

How is Anisotropy introduced? An example:

- Introduction of additional branes: Lifhsitz-like Supergravity solutions
  $ds^2 = r^{2z}(dt^2 + dx_i^2) + r^2 dx_3^2 + \frac{dr^2}{r^2}$. *(Azeyanagi, Li, Takayanagi, 2009)*

	x_0	x_1	x_2	x_3	u	S^5
D3	x	x	x	x		
D7	x	x	x			x

- Which equivalently leads to the following deformation diagram.



It is equivalent to consider the IIB supergravity

$$S = \frac{1}{2\kappa_{10^2}} \int d^{10}x \sqrt{-g} \left[e^{-2\phi}(R + 4\partial_M\phi\partial^M\phi) - \frac{1}{2}F_1^2 - \frac{1}{4\cdot 5!}F_5^2 \right], \quad F_1 := d\chi\,.$$

where $M = 0, \ldots, 9$ and F_1 is the axion field-strength. The equations of motion that the background solves are:

$$R + 4g^{MN}\left(\nabla_M\nabla_N\phi - \partial_M\phi\partial_N\phi\right) = 0\,,$$

$$R_{MN} + 2\nabla_M\nabla_N\phi + \frac{1}{4}g_{MN}e^{2\phi}\partial_P\chi\partial^P\chi - \frac{1}{2}e^{2\phi}\left(F_M F_N + \frac{1}{48}F_{MABCD}F_N{}^{ABCD}\right) = 0\,.$$

plus the Bianchi identities and self duality constraints. The axion field equation is satisfied trivially for linear axion.

The anisotropic IIB solution

The metric in string frame *(Mateos, Trancanelli, 2011)*

$$ds^2 = \frac{1}{u^2}\left(-\mathcal{F}\mathcal{B}\, dx_0^2 + dx_1^2 + dx_2^2 + \mathcal{H} d{x_3}^2 + \frac{du^2}{\mathcal{F}}\right) + \mathcal{Z}\, d\Omega_{S^5}^2 \,.$$

The functions $\mathcal{F}, \mathcal{B}, \mathcal{H}$ depend on the radial direction u and the anisotropy. The anisotropic parameter is α with units of inverse length $(\chi = \alpha x_3)$.
In sufficiently high temperatures, $T \gg \alpha$, the Einstein equations can be solved analytically:

$$\mathcal{F}(u) = 1 - \frac{u^4}{u_h^4} + \alpha^2 \frac{1}{24 u_h^2}\left[8u^2(u_h^2 - u^2) - 10u^4 \log 2 + (3u_h^4 + 7u^4)\log\left(1 + \frac{u^2}{u_h^2}\right)\right]$$

$$\mathcal{B}(u) = 1 - \alpha^2 \frac{u_h^2}{24}\left[\frac{10u^2}{u_h^2 + u^2} + \log\left(1 + \frac{u^2}{u_h^2}\right)\right], \quad \mathcal{H}(u) = \left(1 + \frac{u^2}{u_h^2}\right)^{\frac{\alpha^2 u_h^2}{4}} .$$

The isotropic limit $\alpha \to 0$ reproduce the well know result of the isotropic D3-brane solution (dual to $\mathcal{N} = 4$ finite Temperature sYM solution).

Some Properties of the solution:

- UV: Asymptotically AdS.

$$ds^2 = \frac{1}{u^2}\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + du^2\right)$$

- IR: Lifshitz-like anisotropic theory with fixed scaling $z = \frac{3}{2}$.

$$ds^2 = \frac{1}{u^2}\left(-f(u)dt^2 + dx_1^2 + dx_2^2 + \frac{du^2}{f(u)}\right) + u^{-2/z}dx_3^2$$

- The pressure inequality $P_{x_3} < P_{x_1 x_2}$ does not follow the way that the gravity is deformed $g_{33} \lessgtr g_{11}$.

Questions:

- Why the IR solution is with fixed scale z?
- Is there any solution to flow to a geometry with arbitrary scaling z?

New Anisotropic Theories

Consider a generalized Einstein-Axion-Dilaton action with a potential for the dilaton and an arbitrary coupling between the axion and the dilaton:

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{1}{2} Z(\phi)(\partial\chi)^2 \right] .$$

The eoms read

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2} Z(\phi)\partial_\mu\chi\partial_\nu\chi - \frac{1}{4} g_{\mu\nu}(\partial\phi)^2 - \frac{1}{4} g_{\mu\nu} Z(\partial\chi)^2 + \frac{1}{2} g_{\mu\nu} V(\phi) ,$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g} g^{\mu\nu}\partial_\nu\phi\right) = \frac{1}{2}\partial_\phi Z(\phi)(\partial\chi)^2 - V'(\phi) ,$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g} g^{\mu\nu}\partial_\nu\chi\right) = 0 .$$

Note:
a) The last equation is satisfied for linear axion $\chi = \alpha x_3$.
b) The RHS of the Einstein eoms for radial dependent dilaton $\phi = \phi(u)$ satisfy

$$E_{11} = E_{22} = E_{33} + \left(-\frac{1}{2} Z(\phi)\partial_3\chi\partial_3\chi + \ldots\right) .$$

Let us apply the black hole background ansatz

$$ds^2 = \frac{e^{-\frac{1}{2}\phi(u)}}{u^2}\left(-\mathcal{F}\mathcal{B}\,dt^2 + dx_1^2 + dx_2^2 + \mathcal{H}dx_3^2 + \frac{du^2}{\mathcal{F}}\right),$$
$$\chi = \alpha x_3\,, \qquad \phi = \phi(u)\,,$$

$\phi(u), \mathcal{B}(u), \mathcal{F}(u), \mathcal{H}(u)$ four functions to be found, and α is the constant anisotropic parameter.

How to solve the system?

Solving the system

- Expanding the eoms near the horizon and solving order by order

$$\phi(u) = \phi_h + \phi_1(u-u_h) + \phi_2(u-u_h)^2 + \phi_3(u-u_h)^3 + \mathcal{O}(u^4) \ ,$$
$$\mathcal{H}(u) = \mathcal{H}_h + \mathcal{H}_1(u-u_h) + \mathcal{H}_2(u-u_h)^2 + \mathcal{H}_3(u-u_h)^3 + \mathcal{O}(u^4) \ ,$$

$\phi_i, \mathcal{H}_i$ are solved w.r.t. $(\phi_h, \mathcal{H}_h, V_h, Z_h)$.

- The functions $\mathcal{F}, \mathcal{B}$ are expressed in terms of $\phi_i, \mathcal{H}_i$.

- The boundary conditions are isotropic AdS where regularity at the black hole horizon is imposed $\rightarrow$ All the horizon data is fixed in terms of the scales of the theory!

- Let us focus in the deconfined phase: The Anisotropy and Temperature (α, $T \sim u_h$) are the scales of the theory.

Numerical Solutions: A demonstration

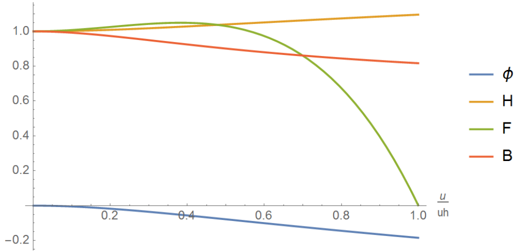
- It's time to fix all the functions

$$V(\phi) = 12\cosh(\sigma\phi) - 6\sigma^2\phi^2, \qquad Z(\phi) = e^{2\gamma\phi},$$

The potential inspired by *((Gubser, Nellore), Pufu, Rocha 2008a,b)* .

- Fixing (γ, σ) and α and u_h we get the metric flow from boundary to horizon:

$$ds^2 = \frac{e^{-\frac{1}{2}\phi(u)}}{u^2}\left(-\mathcal{F}\mathcal{B}\,dt^2 + dx_1^2 + dx_2^2 + \mathcal{H}dx_3^2 + \frac{du^2}{\mathcal{F}}\right),$$



- For $\gamma = 1, \sigma = 0$ we get the IIB supergravity action and the corresponding MT solution. *(Mateos, Trancanelli, 2011)*

IR Solutions

- We study the IR limit vs the potential and the axion-dilaton coupling:

$$V(\phi) = 12\cosh(\sigma\phi) - 6\sigma^2\phi^2 \ , \qquad Z(\phi) = e^{2\gamma\phi} \ .$$

- For $\sigma = 0$ we obtain a IR Lifshitz-like solution:

$$ds^2 = -u^{2z}\left(f(u)dt^2 + dx_i^2\right) + \tilde{\alpha}u^2 dx_3^2 + \frac{du^2}{f(u)u^2} \ , \quad z = \frac{1+2\gamma^2}{2\gamma^2} \ .$$

Note: For $\gamma = 1 \Rightarrow z = 3/2$, we obtain the supergravity MT space. The parameter γ controls the scaling z. This is why the supergravity backgrounds have it fixed.

- For $\sigma \neq 0$ we get an IR hyperscaling violation anisotropic solution

$$ds^2 = u^{-\frac{2\theta}{d}}\left(-u^{2z}\left(f(u)dt^2 + dx_i^2\right) + \tilde{\alpha}u^2 dx_3^2 + \frac{du^2}{f(u)u^2}\right) \ ,$$

with

$$z = \frac{2+4\gamma^2-3\sigma^2}{2\gamma(2\gamma-3\sigma)} \ , \quad \theta = \frac{3\sigma}{2\gamma} \ , \quad f(u) = 1-\left(\frac{u_h}{u}\right)^{1+3z-\theta} \ , \quad T = \frac{|1+3z-\theta|}{4\pi}u_h^z$$

We have obtained the theories, are all of them physical and stable?

Null Energy Condition

- The averaged radial acceleration between two null geodesics is

$$A_r = -4\pi T_{\mu\nu} N^\mu N^\nu \ ,$$

if it is negative the null geodesics observe a non-repulsive gravity on nearby particles along them.

- This imposes the Null Energy Condition

$$T_{\mu\nu} N^\mu N^\nu \geq 0 \ , \quad N^\mu N_\mu = 0 \ ,$$

leading to the following constrains:

- For the Lifshitz-like space $z \geq 1$.
- For the Hyperscaling violation anisotropic metric in 3+1-dim spacetime and anisotropic in 1-dim reads

$$(z-1)(1-\theta+3z) \geq 0 \ ,$$
$$\theta^2 - 3 + 3z(1-\theta) \geq 0 \ .$$

Additional conditions from thermodynamics?

Local Thermodynamic Stability

- The necessary and sufficient conditions for local thermodynamical stability in the canonical ensemble are

$$c_\alpha = T\left(\frac{\partial S}{\partial T}\right)_\alpha \geq 0 \;, \qquad \Phi' = \left(\frac{\partial \Phi}{\partial \alpha}\right)_T \geq 0$$

  c_α is the specific heat: increase of the temperature leads to increase of the entropy.
  Φ' is derivative of the potential: the system is stable under infinitesimal charge fluctuations.

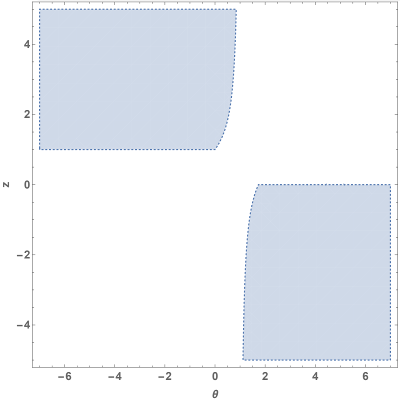
- In the GCE these conditions should be equivalent of having no positive eigenvalues of the Hessian matrix of the entropy with respect to the thermodynamic variables. *(Gubser, Mitra 2001)*

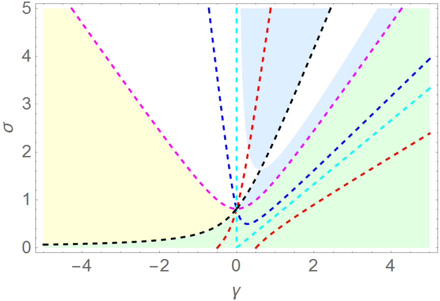
- In the IR the positivity of the specific heat imposes

$$c_\alpha = 1 - \theta + 2z \geq 0$$

Three conditions that constrain (z, θ) and as a result (γ, σ).

$$(z-1)(1-\theta+3z) \geq 0 \ ,$$
$$\theta^2 - 3 + 3z(1-\theta) \geq 0 \ ,$$
$$1-\theta+2z \geq 0 \ .$$





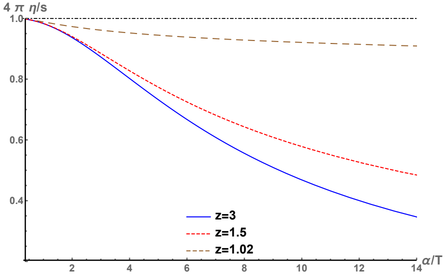
NEC:blue, c_α:yellow, allowed region:green

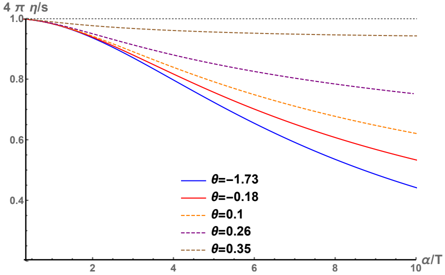
Transport and Diffusion

- For the shear viscosity η we need to solve the system of metric fluctuations on the anisotropic theory. The system can be mapped under a toroidal compactification to a Maxwell system with a mass term
$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(-\frac{1}{4g_{eff}^2}F^2 - \frac{1}{4}m^2(u)A^2\right) ,$$
where A is related to metric fluctuations.
- It can be solved at the zero momentum limit, and AdS asymptotics using the regularity of the horizon i.e. the Eddington-Finklestein coordinate dependence.
- The result is
$$\frac{\eta_{13}}{s} = \frac{1}{4\pi}\frac{g_{11}}{g_{33}}\Big|_{u=u_h}$$
Therefore for prolate geometries $(g_{33} > g_{11} = g_{22})$ is becoming parametrically less than the KSS prediction$(1/4\pi)$!

η/s in our theory

The shear viscosity over entropy ratio for arbitrary (z, θ).





The ratio depends on the temperature as

$$\frac{\eta_{13}}{s} \sim \left(\frac{T}{\tilde{\alpha}|1+3z-\theta|} \right)^{2-\frac{2}{z}} .$$

- For Lifshtiz-like theories ($\theta = 0$) the range of the temperature power is $[0, 2)$.
- For hyperscaling violation theories the range of the temperature power is $[0, \infty)$.

Conclusions

- ✓ We have presented new black hole gravity dual solutions that in the UV are AdS isotropic, while in the IR flow to Lifshitz and hyperscaling violation anisotropic solutions with arbitrary exponents.
- ✓ There are certain stability conditions that constrain the parameters of the backgrounds.
- ✓ The Shear viscosity over entropy density ratio, takes values parametrically lower than $1/4\pi$, and depends on the Temperature as $T^{2-2/z}$.
- Analytic solutions for low α/T.
- Several ways to probe the theory (Mesons, Energy loss of Quarks, Diffusion of Quarks, Speed of Sound, Entanglement Entropy...) and compare with the fixed scaling z solutions.

(D.G 2012)

- Confining (IIB Solitonic) Anisotropic Theories? Hawking-Page transitions expected to depend on the anisotropy $\rightarrow$ Richer Phase diagram! We will report soon on them.

Thank you

Thermodynamics

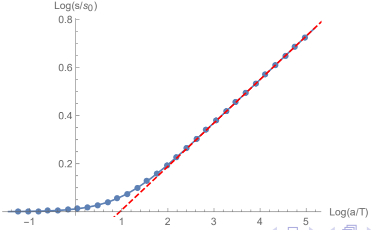
- The entropy density is proportional to the area of the horizon from the Bekenstein-Hawking formula.
- In the hyperscaling anisotropic metric

$$s \sim r_h^{c_1} \sim t^{\frac{-c_1}{z}} \sim T^{\frac{c_1}{z}} \ , \qquad c_1 := 1 - \theta + 2z \ .$$

Thermodynamically the theory 'behaves' as being in the $2 + (1-\theta)/z$ isotropic dimensions.
- In UV the AdS black brane has

$$s \sim T^3$$

- The entropy density for the anisotropic flow ($\gamma = 2, \sigma = 1$):



Transport and Diffusion-Analytic

To compute the shear viscosity η we need to solve the system of metric fluctuations on the anisotropic theory.

$$ds^2 = g_{\mu\nu} dX^\mu dX^\nu + g_{33}(dx_3 + A_\mu dX^\mu)(dx_3 + A_\nu dX^\nu)$$
$$= (g_{\mu\nu} + g_{33} A_\mu A_\nu) dx^\mu dx^\nu + 2 g_{33} A_\nu dx^3 dx^\nu + g_{33}(dx^3)^2$$

where none of the metric elements depend on x_3 and $\mu, \nu \neq 3$. Then the relevant part of the action is The problem can be mapped under a toroidal compactification to a Maxwell system with a mass term

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(-\frac{1}{4 g_{eff}^2} F^2 - \frac{1}{4} m^2(u) A^2 \right) ,$$

where

$$2k := \log(g_{33}) , \qquad \phi_d = \phi - \frac{k}{2} , \qquad \frac{L_3}{2\kappa_5^2} := \frac{1}{2\kappa^2} ,$$
$$m^2(u) = Z(\phi_d + \frac{k}{2}) A_\mu A^\mu (\partial_z \chi)^2 , \quad \frac{1}{g_{eff}^2} = (g_{33}(u))^{3/2} .$$

Before we study the answer, lets mention some other anisotropic examples:

- The bottom-up construction , *(Janik, Witaszczyk, 2008)*

$$ds^2 = \frac{1}{u^2}\left(-a(u)dt^2 + b(u)dx_{3,L}^2 + c(u)\left(dx_{1,T}^2 + dx_{2,T}^2\right) + du^2\right) \ ,$$

  where $a(u), b(u), c(u)$ functions are specified. With an advantage of $P_L < P_T \Rightarrow b(u) < c(u)$.

- The system with a gravity, massless scalar field ϕ, and cosmological constant in 5 dim *(Jain, Kundu, Sen, Sinha, Trivedi, 2014)*

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(R + 12\Lambda - \frac{1}{2}(\partial\phi)^2\right) \ ,$$

  has a solution of the form

$$ds^2 = -A(u)dt^2 + \frac{du^2}{A(u)} + B(u)(dx_1^2 + dx_2^2) + C(u)dx_3^2 \ ,$$

  with $\phi = kx_3$.

- Lifshitz-like Vaidya backgrounds with arbitrary z, have been found, and used to study the thermalization.
  ((Ageev), Arefeva, Golubtsova, Gourgoulhon 2016a,b))