Feynman Rules for the Standard Model Effective Field Theory in R_ξ-gauges

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Talk based on a work in progress with
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Standard Model Effective Field Theory (SMEFT)

Let SM be an effective field theory (EFT), describing the low-energy limit of a more fundamental one (UV) living at a scale Λ.

Then at low-energies,

$$\mathcal{L} = \mathcal{L}_{\text{S}M}^{(4)} + \left(\frac{c^{(5)}}{\Lambda}\right) Q^{(5)} + \left(\frac{c^{(6)}}{\Lambda^2}\right) Q^{(6)} + \dots ,$$

where all operators in terms of SM-fields only.

The more we know the details of the UV-theory (GUTs, MSSM, *etc.*) the more the Wilson coefficients c's are constrained (UV-symmetries, masses, couplings, etc.) or even determined.

- What if we don't know the UV-theory...

We "simply" consider all possible operators with SM content (symmetry-fields).

Warsaw basis

$$\mathcal{L} = \mathcal{L}_{\rm SM}^{(4)} + C^{\nu\nu} Q^{\nu\nu} + \sum_X C^X Q^X + \mathcal{O}(\Lambda^{-3}) \; , \quad ^\star$$

- Complete, irreducible, basis of $SU(3) \times SU(2) \times U(1)$ operators (SM-fields) up to dim-6.
- $X = \{1, ..., 59 + 4\}$ operators.
- But SSB not considered...

**for convenience we have absorbed the* Λ *scale, i.e.,* $C^{\nu\nu} = c^{\nu\nu}/\Lambda, \; C^X = c^X/\Lambda^2$*)*

When SSB is considered some technical complications arise:

- Kinetic terms typically become non-canonical.
- Mixed field strength operators create bilinear kinetic mixing.
- In R_ξ gauge there are many bilinears.

Several attempts have been made (non-linear gauges, mixed propagators, different masses for ghosts, goldstones, etc.)

We consider SMEFT in the presence of electroweak symmetry breaking,

- in the most familiar class of linear R_ξ-gauges,
- we obtain SM-like propagators for all fields,
- we obtain a ghost sector which preserves the BRST invariance of the SMEFT action,
- we derive a full set of Feynman Rules in *mass basis* with $\xi_W, \xi_Z, \xi_A, \xi_G$ for better cross checks.

Always up to $\mathcal{O}(\Lambda^{-3})$ corrections

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Pattern of SSB

SMEFT symmetries/representations = SM symmetries/representations

$$SU(3) \times SU(2) \times U(1)_Y \rightarrow SU(3) \times U(1)_{em}$$

$$Q = T^3 + Y$$

$$\varphi = \begin{pmatrix} \Phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\Phi^0) \end{pmatrix} ,$$

$$H = \text{Higgs}$$

$$\Phi^+ \equiv (\Phi^-)^*,\ \Phi^0 = \text{would-be-Goldstones}$$

Pattern of EW-breaking is unaffected.
However, contrary to SM these are **not mass basis** fields!

Neutral Goldstones

Focus on the neutral Goldstone terms of the theory:

$$\begin{aligned}\mathcal{L}_{\mathrm{G}^0}^{\mathsf{Bilinear}} &\subset (D_\mu\varphi)^\dagger(D^\mu\varphi) + C^{\varphi D}(\varphi^\dagger D_\mu\varphi)^*(\varphi^\dagger D^\mu\varphi) \\ &= \frac{1}{2}\left(1+\frac{1}{2}C^{\varphi D}v^2\right)(\partial_\mu\Phi^0)^2.\end{aligned}$$

the kinetic term is clearly non-canonical. Rescaling as

$$G^0 = Z_{G^0}\,\Phi^0\ ,$$

with the constant factor:

$$Z_{G^0} \equiv 1+\frac{1}{4}C^{\varphi D}v^2\ ,$$

results in **canonical** kinetic terms for the **mass basis** field G^0,

$$\mathcal{L}_{\mathrm{G}^0}^{\mathsf{Bilinear}} = \frac{1}{2}(\partial_\mu G^0)^2.$$

Analogous rescaling for $h = Z_h H$ and other fields in the theory...

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Relevant operators for EW gauge field bilinears are:

$$\mathcal{L}_{\rm EW} = -\frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\varphi)^\dagger(D^\mu\varphi)$$
$$+ C^{\varphi W}(\varphi^\dagger\varphi)W^I_{\mu\nu}W^{I\mu\nu} + C^{\varphi B}(\varphi^\dagger\varphi)B_{\mu\nu}B^{\mu\nu} + C^{\varphi WB}(\varphi^\dagger\tau^I\varphi)W^I_{\mu\nu}B^{\mu\nu}$$
$$+ C^{\varphi D}(\varphi^\dagger D_\mu\varphi)^*(\varphi^\dagger D^\mu\varphi)$$

and obviously only the linear part of $W_{\mu\nu}$ is relevant.

Clearly, for

$$<\varphi>= \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

non-canonical kinetic terms appear.

In general, the rescalings:

$$\begin{aligned} \bar{W}^I_\mu &\equiv Z_g W^I_\mu, & \bar{g} &\equiv Z_g^{-1} g \ , \\ \bar{B}_\mu &\equiv Z_{g'} B_\mu \ , & \bar{g}' &\equiv Z_{g'}^{-1} g', \end{aligned}$$

with constant factors respect gauge-invariance. They satisfy (V = Vector field),

$$g V_\mu = \bar{g} \bar{V}_\mu$$

and thus,

$$D_\mu^{EW} = \bar{D}_\mu^{EW} = \partial_\mu + i\bar{g}\bar{B}_\mu Y + i\bar{g}\bar{W}^I_\mu T^I$$

But if we choose in addition the suitable value,

$$Z_g \equiv 1 - C^{\varphi W} v^2 ,$$
$$Z_{g'} \equiv 1 - C^{\varphi B} v^2$$

we can render EW-gauge bosons canonical, up to one mixed field-strength operator

$$\mathcal{L}_{EW}^{\text{Bilinear}} = -\frac{1}{4}\bar{W}^I_{\mu\nu}\bar{W}^{I\mu\nu} - \frac{1}{4}\bar{B}_{\mu\nu}\bar{B}^{\mu\nu} + C^{\varphi WB}(\varphi^\dagger\tau^I\varphi)\bar{W}^I_{\mu\nu}\bar{B}^{\mu\nu}$$
$$+ (\bar{D}_\mu\varphi)^\dagger(\bar{D}^\mu\varphi) + C^{\varphi D}(\varphi^\dagger\bar{D}_\mu\varphi)^*(\varphi^\dagger\bar{D}^\mu\varphi) .$$

For $\varphi =< \varphi >$ only τ^3 is relevant. Thus,

- only neutral EW-sector affected by the mixed bilinear $\propto C^{\varphi WB}$

The neutral electroweak sector, reads

$$\begin{aligned}\mathcal{L}_{EW}^{\text{Bilinear}} &= -\frac{1}{4}\begin{pmatrix}\bar{W}^3_{\mu\nu}\\ \bar{B}_{\mu\nu}\end{pmatrix}^{\top}\begin{pmatrix}1 & \epsilon\\ \epsilon & 1\end{pmatrix}\begin{pmatrix}\bar{W}^{3\mu\nu}\\ \bar{B}^{\mu\nu}\end{pmatrix}\\ &+\frac{v^2}{8}Z_{G^0}^2\begin{pmatrix}\bar{W}^3_{\mu}\\ \bar{B}_{\mu}\end{pmatrix}^{\top}\begin{pmatrix}\bar{g}^2 & -\bar{g}\bar{g}'\\ -\bar{g}\bar{g}' & \bar{g}'^2\end{pmatrix}\begin{pmatrix}\bar{W}^{3\mu}\\ \bar{B}^{\mu}\end{pmatrix},\end{aligned}$$

where we have defined,

$$\epsilon \equiv C^{\varphi WB}\, v^2 \,.$$

The congruent matrix transformation,

$$\begin{pmatrix}\bar{W}^3_{\mu}\\ \bar{B}_{\mu}\end{pmatrix} = \mathbb{X}\begin{pmatrix}Z_{\mu}\\ A_{\mu}\end{pmatrix},$$

with the matrix $\mathbb{X}$ taking the form,

$$\mathbb{X} = \begin{pmatrix}1 & -\frac{\epsilon}{2}\\ -\frac{\epsilon}{2} & 1\end{pmatrix}\begin{pmatrix}\cos\bar{\theta} & \sin\bar{\theta}\\ -\sin\bar{\theta} & \cos\bar{\theta}\end{pmatrix}.$$

results in **canonical kinetic terms** in mass basis (Z_μ, A_μ), and diagonal masses:

$$\begin{aligned}M_Z &= \frac{1}{2}\sqrt{\bar{g}^2+\bar{g}'^2}\,v\left(1+\frac{\epsilon\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}\right)Z_{G^0}\,,\\ M_A &= 0\,.\end{aligned}$$

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Gauge-Goldstone mixing

$$\mathcal{L}_H \supset \ (\bar{D}_\mu \varphi)^\dagger (\bar{D}^\mu \varphi) \ + \ C^{\varphi D} (\varphi^\dagger \bar{D}_\mu \varphi)^* (\varphi^\dagger \bar{D}^\mu \varphi) \ .$$

SSB generates some strange "unwanted" terms, which however in mass basis (i.e., apply $\mathbb{X}, Z_G^0$), are simply

$$\mathcal{L}_{G-EW} \quad = \quad i M_W (W_\mu^+ \partial^\mu G^- - W_\mu^- \partial^\mu G^+) - M_Z \, Z_\mu \, \partial^\mu G^0 \ ,$$

- **In *mass basis* all G-EW *"Wilsons"* become absorbed in fields and masses.**

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Gauge fixing

Our starting point is:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{GF} + \mathcal{L}_{FP}$$

with

$$\mathcal{L}_0 = \mathcal{L}^{(4)}_{\mathsf{SM}} + \mathcal{L}^{(5)}_{\mathsf{SMEFT}} + \mathcal{L}^{(6)}_{\mathsf{SMEFT}}$$

expressed in terms of barred fields and couplings.

- QCD sector as in SM
- But EW sector ...

Our choice for a Gauge-Fixing (GF) term is:

$$\mathcal{L}_{GF} = -\frac{1}{2} \boldsymbol{F}^\top \hat{\boldsymbol{\xi}}^{\mathbf{-1}} \boldsymbol{F}$$

with the gauge fixing functions F^i defined through

$$\boldsymbol{F} = \begin{pmatrix} F^1 \\ F^2 \\ F^3 \\ F^0 \end{pmatrix} = \begin{pmatrix} \partial_\mu \bar{W}^{1\mu} \\ \partial_\mu \bar{W}^{2\mu} \\ \partial_\mu \bar{W}^{3\mu} \\ \partial_\mu \bar{B}^\mu \end{pmatrix} - \frac{v\hat{\boldsymbol{\xi}}}{2} \begin{pmatrix} -i\bar{g}\frac{\Phi^+ - \Phi^-}{\sqrt{2}} \\ \bar{g}\frac{\Phi^+ + \Phi^-}{\sqrt{2}} \\ -\bar{g} Z_{G^0}^2 \Phi_0 \\ \bar{g}' Z_{G^0}^2 \Phi_0 \end{pmatrix}$$

and a 4×4 *symmetric* matrix $\hat{\xi}$, introduced as

$$\hat{\boldsymbol{\xi}} = \begin{pmatrix} \xi_W & & & 0 \\ & \xi_W & & \\ 0 & & \mathbb{X} \begin{pmatrix} \xi_Z & \\ & \xi_A \end{pmatrix} \mathbb{X}^\top & \end{pmatrix},$$

with $\mathbb{X}$ being the 2×2 mixing matrix of the neutral electroweak gauge bosons.

Chosen in such a way that ...

Gauge-fixing in mass basis

in **mass basis**, ($i.e.$, $\mathbb{X}$, Z^0_G, $W^\pm_\mu = (\bar{W}^1_\mu \mp i\bar{W}^2_\mu)/\sqrt{2}$):

$$\begin{aligned}\mathcal{L}_{GF} = \quad & - \quad \frac{1}{\xi_W}(\partial^\mu W^+_\mu + i\xi_W M_W G^+)(\partial^\nu W^-_\nu - i\xi_W M_W G^-) \\ & - \quad \frac{1}{2\xi_Z}(\partial^\mu Z_\mu + \xi_Z M_Z G^0)^2 - \frac{1}{2\xi_A}(\partial^\mu A_\mu)^2\end{aligned}$$

which is identical to SM in the standard linear R_ξ-gauge.

That is,

- Goldstones $G^\pm, G^0$are canonical with squared masses: $\xi_W M_W^2, \xi_Z M_Z^2$.
- Goldstone propagators are SM-like (we see later).
- Unwanted gauge-Goldstone bilinear mixing vanishes (total derivative).
- ξ-dependent part of gauge kinetic, as in SM (diagonal).

Convenient and **consistent** choice:

$$\mathcal{L}_{FP} = \bar{\boldsymbol{N}}^\top \hat{\boldsymbol{E}}(\hat{\boldsymbol{M}}_{\boldsymbol{F}} \boldsymbol{N})$$

where (gauge basis),

$$\text{ghosts:}\ N^i = (N^1, N^2, N^3, N^0)$$

$$\text{anti-ghosts:}\ \bar{N}^i = (\bar{N}^1, \bar{N}^2, \bar{N}^3, \bar{N}^0)$$

,

and we have introduced the *symmetric* 4×4 matrix,

$$\hat{\boldsymbol{E}} = \begin{pmatrix} \mathbb{I}_{2\times 2} & 0_{2\times 2} \\ 0_{2\times 2} & (\mathbb{X}^\top)^{-1}\mathbb{X}^{-1} \end{pmatrix}.$$

and $\hat{M}_F$ is the Fadeev-Popov (FP) field-dependent matrix for SMEFT, obtained through:

$$\hat{M}_F^{ij} N^j = \boldsymbol{s} F^i, \qquad (\{i, j\} = 1, ..., 4).$$

and $\boldsymbol{s}$ is the BRST-operator which transforms the fields in $F^i(\varphi, \varphi^\dagger, B_\mu, W^I_\mu)$, **exactly as in SM.**

Mass basis FP-ghosts

Convenient because in **mass basis**:

$$\frac{1}{\sqrt{2}}(N^1 \mp iN^2) = \eta^{\pm}, \quad \frac{1}{\sqrt{2}}(\bar{N}^1 \pm i\bar{N}^2) = \bar{\eta}^{\pm}$$

$$\begin{pmatrix} N^3 \\ N^0 \end{pmatrix} = \mathbb{X} \begin{pmatrix} \eta^Z \\ \eta^A \end{pmatrix}, \quad \begin{pmatrix} \bar{N}^3 \\ \bar{N}^0 \end{pmatrix}^{\top} = \begin{pmatrix} \bar{\eta}^Z \\ \bar{\eta}^A \end{pmatrix}^{\top} \mathbb{X}^{\top}$$

- Ghosts are canonical with diagonal squared SM-masses: $\xi_W M_W^2,\ \xi_Z M_Z^2,\ 0$.
- Again, the ghost propagators are SM-like (we see later).

But corrections appear explicitly in ghost vertices (new terms in the SM ghost-vertices).

Consistent because of **BRST-invariance**.

BRST Invariance of $\mathcal{L}_0$

We first neglect the gauge fixing and ghost terms. In the remaining part of the Lagrangian, the $\boldsymbol{s}$-operator acts on all fields as a special gauge transformation with

$$SU(2) : \theta^I = \bar{g} N^I,$$

$$U(1) \; : \theta^0 = \bar{g}' N^0.$$

Hence,

- as in SM, gauge invariance → BRST invariance for $\mathcal{L}_0$:

$$\boldsymbol{s}\mathcal{L}_0 = 0$$

BRST on ghosts

- BRST on ghosts exactly as in SM. Thus as in SM,

$$\boldsymbol{s}^2\{N^i, W^I_\mu, B_\mu, \varphi, \psi\} = 0$$

$$\boldsymbol{s}^2(F^i) = \boldsymbol{s}(M_F^{ij} N^j) = 0 \ .$$

- Anti-ghosts as in SM, up to constant matrices ($i.e.$, $\mathbb{X}$ implicit in $\hat{\xi}^{-1}, \hat{E}^{-1}$).

$$\boldsymbol{s}\bar{N}^i = F^j(\hat{\boldsymbol{\xi}}^{\boldsymbol{-1}}\hat{\boldsymbol{E}}^{\boldsymbol{-1}})^{ji}$$

BRST Invariance of $\mathcal{L}$

Now, using

$$\begin{aligned} \hat{\xi} = \hat{\xi}^T \quad &, \quad \boldsymbol{s}F = \hat{M}_F N, \\ F^\top \hat{\xi}^{-1} = \boldsymbol{s}\bar{N}^\top \hat{E} \quad &, \quad \boldsymbol{s}(M_F N) = 0, \end{aligned}$$

it becomes straightforward to prove:

$$\begin{aligned} \boldsymbol{s}\,\mathcal{L}_{GF} &= -\frac{1}{2}\boldsymbol{s}\Big(F^\top \hat{\xi}^{-1} F\Big) = -F^\top \hat{\xi}^{-1}(\boldsymbol{s}F) = -F^\top \hat{\xi}^{-1} \hat{M}_F N \\ &= -(\boldsymbol{s}\bar{N}^\top \hat{E})\hat{M}_F N = -\boldsymbol{s}\Big(\bar{N}^\top \hat{E}\hat{M}_F N\Big) = -\boldsymbol{s}\,\mathcal{L}_{FP}. \end{aligned}$$

- The full Lagrangian now is BRST-invariant, satisfying

$$\boldsymbol{s}\mathcal{L} = \boldsymbol{s}(\mathcal{L}_0 + \mathcal{L}_{GF} + \mathcal{L}_{FP}) = 0$$

Summary of gauge fixing

- We have started from:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{GF} + \mathcal{L}_{FP} = \mathcal{L}_0 - \frac{1}{2}\boldsymbol{F}^\top \boldsymbol{\hat{\xi}^{-1}} \boldsymbol{F} + \boldsymbol{\bar{N}}^\top \boldsymbol{\hat{E}}(\boldsymbol{\hat{M}_F} \boldsymbol{N})$$

- We have discussed how Goldstone, ghost and ξ-dependent electroweak bilinears become SM-like, in mass basis.
- We have shown that the unwanted gauge-goldstone bilinear mixing cancels as in SM.
- We have shown that the full Lagrangian (action) is BRST invariant.

$$\boldsymbol{s}\mathcal{L} = 0$$

- We have shown that the BRST on all fields, besides anti-ghosts, is nilpotent.

$$\boldsymbol{s}^2\{N^i, W_\mu^I, B_\mu, \varphi, \psi\} = 0$$

What about anti-ghosts?

BRST of SMEFT

As in SM, $\boldsymbol{s}^2 \bar{N}^i \neq 0$. Not a real problem since one can always introduce auxiliary fields B^i. One can verify that

$$\mathcal{L}_{GF} = -\frac{1}{2}\boldsymbol{F}^\top \hat{\boldsymbol{\xi}}^{-1}\boldsymbol{F} = \boldsymbol{B}^\top \hat{\boldsymbol{E}}\boldsymbol{F} + \frac{1}{2}\boldsymbol{B}^\top \hat{\boldsymbol{E}}\hat{\boldsymbol{\xi}}\hat{\boldsymbol{E}}\boldsymbol{B}\Big|_{EOM}$$

Also, by changing only BRST for anti-ghosts as,

$$\boldsymbol{s}\bar{N}^i = B^i,$$
$$\boldsymbol{s}B^i = 0$$

the BRST-invariance of the SMEFT action remains. However now BRST is nilpotent also for auxiliaries and anti-ghosts, that is

$\boldsymbol{s}^2 = 0$, **for all fields in the theory and**

$\boldsymbol{s}_{\mathsf{SMEFT}} = \boldsymbol{s}_{\mathsf{SM}}$

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At tree-level SMEFT:

- Fermion kinetics unaffected by SSB. But corrections to masses and tree-level FCNC exist.
- CKM, PMNS, not unitary any more, i.e., corrections to the charged currents

$$K_{\rm CKM} \equiv K \equiv U^\dagger_{u_L}(\mathbb{I} + v^2 C'^{\varphi q(3)})U_{d_L} ,$$

$$U_{\rm PMNS} \equiv U \equiv U^\dagger_{e_L}(\mathbb{I} + v^2 C'^{\varphi \ell(3)})U_{\nu_L} .$$

- Only left handed rotations appear in mass basis through CKM, PMNS. All right handed rotations absorbed in a redefinition of *mass basis* Wilsons of the flavor sector.
- The ρ-parameter is anomalous:

$$\rho = \frac{|J_{C.C}|^2}{|J_{N.C.}|^2} = \frac{\bar{g}^2\, M_Z^2}{\bar{g}_Z^2 M_W^2} = 1 + \frac{1}{2} C^{\varphi D} v^2 .$$

  due to the breaking of the custodial symmetry in the Higgs potential, by the relevant Wilson.

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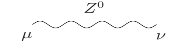
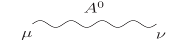
1.1 Propagators in the R_ξ gauge

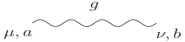
$$-\frac{i}{k^2 - M_W^2}\left[g^{\mu\nu} - (1-\xi_W)\frac{k_\mu k_\nu}{k^2 - \xi_W M_W^2}\right]$$



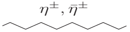
$$-\frac{i}{k^2 - M_Z^2}\left[g^{\mu\nu} - (1-\xi_Z)\frac{k_\mu k_\nu}{k^2 - \xi_Z M_Z^2}\right]$$



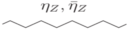
$$-\frac{i}{k^2}\left[g^{\mu\nu} - (1-\xi_A)\frac{k_\mu k_\nu}{k^2}\right]$$



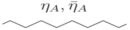
$$-\frac{i\delta_{ab}}{k^2}\left[g^{\mu\nu} - (1-\xi_G)\frac{k_\mu k_\nu}{k^2}\right]$$



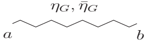
$$-\frac{i}{k^2 - \xi_W M_W^2}$$



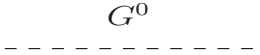
$$-\frac{i}{k^2 - \xi_Z M_Z^2}$$



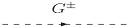
$$-\frac{i}{k^2}$$



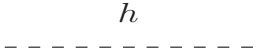
$$-\frac{i\delta_{ab}}{k^2}$$



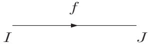
$$\frac{i}{k^2 - \xi_Z M_Z^2}$$



$$\frac{i}{k^2 - \xi_W M_W^2}$$



$$\frac{i}{k^2 - M_h^2}$$



$$\frac{i\delta^{IJ}}{\not{k} - m_f}$$

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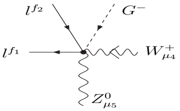
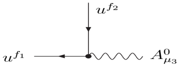
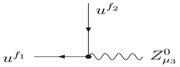


$$+\frac{2\sqrt{2}\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}\sigma^{\mu_4\mu_5}P_L C^{eW*}_{f_2f_1}$$

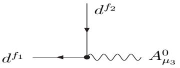
1.4 Quark–gauge vertices



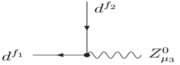
$$\begin{aligned}
&-\frac{2i\bar{g}\bar{g}'}{3\sqrt{\bar{g}^2+\bar{g}'^2}}\delta_{f_1f_2}\gamma^{\mu_3}+\frac{2i\bar{g}^2\bar{g}'^2v^2}{3\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}\delta_{f_1f_2}C^{\phi WB}\gamma^{\mu_3}\\
&-\frac{\sqrt{2}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{uW*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{uW}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)\\
&-\frac{\sqrt{2}\bar{g}v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{uB*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{uB}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)
\end{aligned}$$



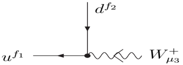
$$\begin{aligned}
&+\frac{i}{6\sqrt{\bar{g}^2+\bar{g}'^2}}\delta_{f_1f_2}\left(\left(\bar{g}'^2-3\bar{g}^2\right)\gamma^{\mu_3}P_L+4\bar{g}'^2\gamma^{\mu_3}P_R\right)\\
&-\frac{i\bar{g}\bar{g}'v^2}{6\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}\delta_{f_1f_2}C^{\phi WB}\left(\left(3\bar{g}'^2-\bar{g}^2\right)\gamma^{\mu_3}P_L-4\bar{g}^2\gamma^{\mu_3}P_R\right)\\
&-\frac{\sqrt{2}\bar{g}v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{uW*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{uW}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)\\
&+\frac{\sqrt{2}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{uB*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{uB}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)\\
&+\frac{1}{2}iv^2\sqrt{\bar{g}^2+\bar{g}'^2}K_{f_1g_2}K^*_{f_2g_1}C^{\phi q1}_{g_2g_1}\gamma^{\mu_3}P_L\\
&-\frac{1}{2}iv^2\sqrt{\bar{g}^2+\bar{g}'^2}K_{f_1g_2}K^*_{f_2g_1}C^{\phi q3}_{g_2g_1}\gamma^{\mu_3}P_L\\
&+\frac{1}{2}iv^2\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi u}_{f_1f_2}\gamma^{\mu_3}P_R
\end{aligned}$$



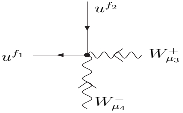
$$
\begin{aligned}
&+\frac{i\bar{g}\bar{g}'}{3\sqrt{\bar{g}^2+\bar{g}'^2}}\delta_{f_1f_2}\gamma^{\mu_3}-\frac{i\bar{g}^2\bar{g}'^2v^2}{3\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}\delta_{f_1f_2}C^{\phi WB}\gamma^{\mu_3}\\
&+\frac{\sqrt{2}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{dW*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{dW}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)\\
&-\frac{\sqrt{2}\bar{g}v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{dB*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{dB}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)
\end{aligned}
$$



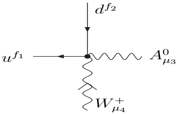
$$
\begin{aligned}
&+\frac{i}{6\sqrt{\bar{g}^2+\bar{g}'^2}}\delta_{f_1f_2}\left(\left(3\bar{g}^2+\bar{g}'^2\right)\gamma^{\mu_3}P_L-2\bar{g}'^2\gamma^{\mu_3}P_R\right)\\
&+\frac{i\bar{g}\bar{g}'v^2}{6\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}\delta_{f_1f_2}C^{\phi WB}\left(\left(\bar{g}^2+3\bar{g}'^2\right)\gamma^{\mu_3}P_L-2\bar{g}^2\gamma^{\mu_3}P_R\right)\\
&+\frac{\sqrt{2}\bar{g}v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{dW*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{dW}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)\\
&+\frac{\sqrt{2}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{dB*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{dB}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)\\
&+\frac{1}{2}iv^2\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi q1}_{f_1f_2}\gamma^{\mu_3}P_L+\frac{1}{2}iv^2\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi q3}_{f_1f_2}\gamma^{\mu_3}P_L\\
&+\frac{1}{2}iv^2\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi d}_{f_1f_2}\gamma^{\mu_3}P_R
\end{aligned}
$$



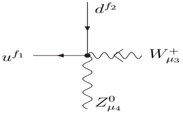
$$
\begin{aligned}
&-\frac{i\bar{g}}{\sqrt{2}}K_{f_1f_2}\gamma^{\mu_3}P_L-2vp_3^{\nu}K_{g_1f_2}\sigma^{\mu_3\nu}P_LC^{uW*}_{g_1f_1}\\
&-2vp_3^{\nu}K_{f_1g_1}C^{dW}_{g_1f_2}\sigma^{\mu_3\nu}P_R-\frac{i\bar{g}v^2}{2\sqrt{2}}C^{\phi ud}_{f_1f_2}\gamma^{\mu_3}P_R
\end{aligned}
$$



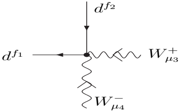
$$
-\sqrt{2}\bar{g}v\left(\sigma^{\mu_3\mu_4}P_LC^{uW*}_{f_2f_1}+C^{uW}_{f_1f_2}\sigma^{\mu_3\mu_4}P_R\right)
$$



$$-\frac{2\bar{g}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}K_{g_1f_2}\sigma^{\mu_3\mu_4}P_L C^{uW*}_{g_1f_1}-\frac{2\bar{g}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}K_{f_1g_1}\sigma^{\mu_3\mu_4}P_R C^{dW}_{g_1f_2}$$

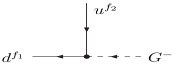


$$+\frac{2\bar{g}^2v}{\sqrt{\bar{g}^2+\bar{g}'^2}}K_{g_1f_2}\sigma^{\mu_3\mu_4}P_L C^{uW*}_{g_1f_1}+\frac{2\bar{g}^2v}{\sqrt{\bar{g}^2+\bar{g}'^2}}K_{f_1g_1}\sigma^{\mu_3\mu_4}P_R C^{dW}_{g_1f_2}$$

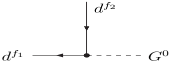


$$+\sqrt{2}\bar{g}v\left(\sigma^{\mu_3\mu_4}P_L C^{dW*}_{f_2f_1}+C^{dW}_{f_1f_2}\sigma^{\mu_3\mu_4}P_R\right)$$

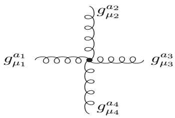
1.5 Quark–Higgs–gauge vertices



$$\begin{aligned}&-\frac{i\sqrt{2}}{v}K^*_{f_2f_1}\Big(m_{d_{f_1}}P_L-m_{u_{f_2}}P_R\Big)\\&+i\sqrt{2}v\left(K^*_{f_2g_2}C^{\phi q3}_{f_1g_2}\Big(m_{d_{f_1}}P_L-m_{u_{f_2}}P_R\Big)-\not{p}_3P_LK^*_{f_2g_1}C^{\phi q3}_{f_1g_1}\right)\\&-\frac{iv}{\sqrt{2}}\not{p}_3P_RC^{\phi ud*}_{f_2f_1}\end{aligned}$$



$$\begin{aligned}&+\frac{1}{v}\delta_{f_1f_2}m_{d_{f_1}}\gamma^5-\frac{v}{4}\delta_{f_1f_2}C^{\phi D}m_{d_{f_1}}\gamma^5\\&-v\not{p}_3P_LC^{\phi q1}_{f_1f_2}-v\not{p}_3P_LC^{\phi q3}_{f_1f_2}-v\not{p}_3P_RC^{\phi d}_{f_1f_2}\end{aligned}$$



$$+i\bar{g}_s^2 \left((\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3} - \eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}) f_{a_1a_2b_1} f_{a_3a_4b_1}\right.$$
$$+ (\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3} - \eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}) f_{a_1a_3b_1} f_{a_2a_4b_1}$$
$$\left.+ (\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4} - \eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}) f_{a_1a_4b_1} f_{a_2a_3b_1}\right)$$
$$- 6i\bar{g}_s C^G \left(f_{a_1a_4b_1} f_{a_2a_3b_1} \left(-p_2^{\mu_4} p_3^{\mu_2} \eta_{\mu_1\mu_3} + \eta_{\mu_2\mu_4} p_1 \cdot p_4 \eta_{\mu_1\mu_3}\right.\right.$$
$$+ \eta_{\mu_2\mu_4} p_2 \cdot p_3 \eta_{\mu_1\mu_3} + p_1^{\mu_4} (p_4^{\mu_3} \eta_{\mu_1\mu_2} - p_4^{\mu_2} \eta_{\mu_1\mu_3}) + p_1^{\mu_3} p_4^{\mu_2} \eta_{\mu_1\mu_4} - p_1^{\mu_2} p_4^{\mu_3} \eta_{\mu_1\mu_4}$$
$$+ p_2^{\mu_4} p_3^{\mu_1} \eta_{\mu_2\mu_3} - p_2^{\mu_1} p_3^{\mu_4} \eta_{\mu_2\mu_3} - p_1^{\mu_3} p_4^{\mu_1} \eta_{\mu_2\mu_4} + p_2^{\mu_3} (p_3^{\mu_4} \eta_{\mu_1\mu_2} - p_3^{\mu_1} \eta_{\mu_2\mu_4})$$
$$\left.+ p_2^{\mu_1} p_3^{\mu_2} \eta_{\mu_3\mu_4} + p_1^{\mu_2} p_4^{\mu_1} \eta_{\mu_3\mu_4} - \eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4} p_1 \cdot p_4 - \eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4} p_2 \cdot p_3\right)$$
$$+ f_{a_1a_3b_1} f_{a_2a_4b_1} \left(p_1^{\mu_4} p_3^{\mu_2} \eta_{\mu_1\mu_3} - p_1^{\mu_2} p_3^{\mu_4} \eta_{\mu_1\mu_3} - p_2^{\mu_3} p_4^{\mu_2} \eta_{\mu_1\mu_4}\right.$$
$$+ p_1^{\mu_3} (p_3^{\mu_4} \eta_{\mu_1\mu_2} - p_3^{\mu_2} \eta_{\mu_1\mu_4}) - p_1^{\mu_4} p_3^{\mu_1} \eta_{\mu_2\mu_3} + p_2^{\mu_4} (p_4^{\mu_3} \eta_{\mu_1\mu_2} - p_4^{\mu_1} \eta_{\mu_2\mu_3})$$
$$+ p_2^{\mu_3} p_4^{\mu_1} \eta_{\mu_2\mu_4} - p_2^{\mu_1} p_4^{\mu_3} \eta_{\mu_2\mu_4} + p_1^{\mu_2} p_3^{\mu_1} \eta_{\mu_3\mu_4} + p_2^{\mu_1} p_4^{\mu_2} \eta_{\mu_3\mu_4}$$
$$\left.+ \eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3} p_1 \cdot p_3 - \eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4} p_1 \cdot p_3 + \eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3} p_2 \cdot p_4 - \eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4} p_2 \cdot p_4\right)$$
$$+ f_{a_1a_2b_1} f_{a_3a_4b_1} \left(p_1^{\mu_2} p_2^{\mu_4} \eta_{\mu_1\mu_3} + p_3^{\mu_4} p_4^{\mu_2} \eta_{\mu_1\mu_3} - \eta_{\mu_2\mu_4} p_1 \cdot p_2 \eta_{\mu_1\mu_3}\right.$$
$$- \eta_{\mu_2\mu_4} p_3 \cdot p_4 \eta_{\mu_1\mu_3} - p_1^{\mu_2} p_2^{\mu_3} \eta_{\mu_1\mu_4} - p_3^{\mu_2} p_4^{\mu_3} \eta_{\mu_1\mu_4} - p_3^{\mu_4} p_4^{\mu_1} \eta_{\mu_2\mu_3}$$
$$+ p_1^{\mu_4} (p_2^{\mu_3} \eta_{\mu_1\mu_2} - p_2^{\mu_1} \eta_{\mu_2\mu_3}) + p_3^{\mu_1} p_4^{\mu_3} \eta_{\mu_2\mu_4} + p_1^{\mu_3} (p_2^{\mu_1} \eta_{\mu_2\mu_4} - p_2^{\mu_4} \eta_{\mu_1\mu_2})$$
$$\left.\left.+ p_3^{\mu_2} p_4^{\mu_1} \eta_{\mu_3\mu_4} - p_3^{\mu_1} p_4^{\mu_2} \eta_{\mu_3\mu_4} + \eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3} p_1 \cdot p_2 + \eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3} p_3 \cdot p_4\right)\right)$$
$$- 2i\bar{g}_s C^{\tilde{G}} \Big(-\epsilon_{\mu_1\mu_2\mu_3\alpha_1} f_{a_1a_2b_1} f_{a_3a_4b_1} p_1^{\mu_4} p_2^{\alpha_1} + \epsilon_{\mu_2\mu_3\mu_4\alpha_1} f_{a_1a_4b_1} f_{a_2a_3b_1} p_3^{\mu_1} p_2^{\alpha_1}$$
$$- \epsilon_{\mu_1\mu_2\mu_3\alpha_1} f_{a_1a_4b_1} f_{a_2a_3b_1} p_3^{\mu_4} p_2^{\alpha_1} - \epsilon_{\mu_2\mu_3\mu_4\alpha_1} f_{a_1a_3b_1} f_{a_2a_4b_1} p_4^{\mu_1} p_2^{\alpha_1}$$
$$- \epsilon_{\mu_3\mu_4\alpha_1\beta_1} f_{a_1a_4b_1} f_{a_2a_3b_1} p_3^{\beta_1} \eta_{\mu_1\mu_2} p_2^{\alpha_1} + \epsilon_{\mu_3\mu_4\alpha_1\beta_1} f_{a_1a_3b_1} f_{a_2a_4b_1} p_4^{\beta_1} \eta_{\mu_1\mu_2} p_2^{\alpha_1}$$
$$- \epsilon_{\mu_2\mu_4\alpha_1\beta_1} f_{a_1a_4b_1} f_{a_2a_3b_1} p_3^{\beta_1} \eta_{\mu_1\mu_3} p_2^{\alpha_1} - \epsilon_{\mu_2\mu_3\alpha_1\beta_1} f_{a_1a_3b_1} f_{a_2a_4b_1} p_4^{\beta_1} \eta_{\mu_1\mu_4} p_2^{\alpha_1} - \epsilon_{\mu_1\mu_4\alpha_1\beta_1} f_{a_1}$$
$$- \epsilon_{\mu_1\mu_4\alpha_1\beta_1} f_{a_1a_3b_1} f_{a_2a_4b_1} p_4^{\beta_1} \eta_{\mu_2\mu_3} p_2^{\alpha_1} - \epsilon_{\mu_1\mu_3\alpha_1\beta_1} f_{a_1a_4b_1} f_{a_2a_3b_1} p_3^{\beta_1} \eta_{\mu_2\mu_4} p_2^{\alpha_1} - \epsilon_{\mu_1\mu_3\alpha_1\beta_1} f_{a_1}$$
$$- \epsilon_{\mu_1\mu_2\alpha_1\beta_1} f_{a_1a_4b_1} f_{a_2a_3b_1} p_3^{\beta_1} \eta_{\mu_3\mu_4} p_2^{\alpha_1} - \epsilon_{\mu_1\mu_2\alpha_1\beta_1} f_{a_1a_3b_1} f_{a_2a_4b_1} p_4^{\beta_1} \eta_{\mu_3\mu_4} p_2^{\alpha_1} - \epsilon_{\mu_2\mu_3\mu_4\alpha_1} f_{a_1}$$
$$- \epsilon_{\mu_1\mu_2\mu_3\alpha_1} f_{a_1a_2b_1} f_{a_3a_4b_1} p_1^{\alpha_1} p_2^{\mu_4} + \epsilon_{\mu_1\mu_2\mu_3\alpha_1} f_{a_1a_3b_1} f_{a_2a_4b_1} p_1^{\mu_4} p_3^{\alpha_1} + \epsilon_{\mu_2\mu_3\mu_4\alpha_1} f_{a_1a_4b_1} f_{a_2a_3b_1}$$
$$- \epsilon_{\mu_1\mu_2\mu_3\alpha_1} f_{a_1a_4b_1} f_{a_2a_3b_1} p_2^{\mu_4} p_3^{\alpha_1} + \epsilon_{\mu_2\mu_3\mu_4\alpha_1} f_{a_1a_3b_1} f_{a_2a_4b_1} p_1^{\alpha_1} p_3^{\mu_1} + \epsilon_{\mu_1\mu_2\mu_3\alpha_1} f_{a_1a_3b_1} f_{a_2a_4b_1}$$
$$+ \epsilon_{\mu_1\mu_2\mu_3\alpha_1} f_{a_1a_4b_1} f_{a_2a_3b_1} p_1^{\mu_4} p_4^{\alpha_1} - \epsilon_{\mu_2\mu_3\mu_4\alpha_1} f_{a_1a_3b_1} f_{a_2a_4b_1} p_2^{\mu_1} p_4^{\alpha_1} - \epsilon_{\mu_1\mu_2\mu_3\alpha_1} f_{a_1a_3b_1} f_{a_2a_4b_1}$$
$$+ \epsilon_{\mu_2\mu_3\mu_4\alpha_1} f_{a_1a_2b_1} f_{a_3a_4b_1} p_3^{\mu_1} p_4^{\alpha_1} + \epsilon_{\mu_1\mu_2\mu_3\alpha_1} f_{a_1a_2b_1} f_{a_3a_4b_1} p_3^{\mu_4} p_4^{\alpha_1} - \epsilon_{\mu_2\mu_3\mu_4\alpha_1} f_{a_1a_4b_1} f_{a_2a_3b_1}$$
$$+ \epsilon_{\mu_2\mu_3\mu_4\alpha_1} f_{a_1a_2b_1} f_{a_3a_4b_1} p_3^{\alpha_1} p_4^{\mu_1} + \epsilon_{\mu_1\mu_3\mu_4\alpha_1} \left(f_{a_1a_4b_1} f_{a_2a_3b_1} (p_2^{\alpha_1} p_3^{\mu_2} - p_1^{\mu_2} p_4^{\alpha_1} - p_1^{\alpha_1} p_4^{\mu_2})\right.$$
$$\left.+ f_{a_1a_3b_1} f_{a_2a_4b_1} (p_1^{\mu_2} p_3^{\alpha_1} + p_1^{\alpha_1} p_3^{\mu_2} - p_2^{\alpha_1} p_4^{\mu_2}) + f_{a_1a_2b_1} f_{a_3a_4b_1} (p_1^{\mu_2} p_2^{\alpha_1} - p_3^{\mu_2} p_4^{\alpha_1} - p_3^{\alpha_1} p_4^{\mu_2})\right)$$
$$+ \epsilon_{\mu_1\mu_2\mu_4\alpha_1} \left(f_{a_1a_4b_1} f_{a_2a_3b_1} (-p_2^{\mu_3} p_3^{\alpha_1} + p_1^{\mu_3} p_4^{\alpha_1} + p_1^{\alpha_1} p_4^{\mu_3}) + f_{a_1a_3b_1} f_{a_2a_4b_1} (p_1^{\mu_3} p_3^{\alpha_1} - p_2^{\mu_3} p_4^{\alpha_1}\right.$$
$$\left.+ f_{a_1a_2b_1} f_{a_3a_4b_1} (p_1^{\mu_3} p_2^{\alpha_1} + p_1^{\alpha_1} p_2^{\mu_3} - p_3^{\alpha_1} p_4^{\mu_3})\right) - \epsilon_{\mu_3\mu_4\alpha_1\beta_1} f_{a_1a_2b_1} f_{a_3a_4b_1} p_1^{\alpha_1} p_2^{\beta_1} \eta_{\mu_1\mu_2}$$
$$- \epsilon_{\mu_3\mu_4\alpha_1\beta_1} f_{a_1a_3b_1} f_{a_2a_4b_1} p_1^{\alpha_1} p_3^{\beta_1} \eta_{\mu_1\mu_2} + \epsilon_{\mu_3\mu_4\alpha_1\beta_1} f_{a_1a_4b_1} f_{a_2a_3b_1} p_1^{\alpha_1} p_4^{\beta_1} \eta_{\mu_1\mu_2} - \epsilon_{\mu_2\mu_4\alpha_1\beta_1} f_{a_1}$$
$$- \epsilon_{\mu_2\mu_4\alpha_1\beta_1} f_{a_1a_3b_1} f_{a_2a_4b_1} p_1^{\alpha_1} p_3^{\beta_1} \eta_{\mu_1\mu_3} - \epsilon_{\mu_2\mu_4\alpha_1\beta_1} f_{a_1a_4b_1} f_{a_2a_3b_1} p_1^{\alpha_1} p_4^{\beta_1} \eta_{\mu_1\mu_3} + \epsilon_{\mu_2\mu_4\alpha_1\beta_1} f_{a_1}$$
$$+ \epsilon_{\mu_2\mu_3\alpha_1\beta_1} f_{a_1a_2b_1} f_{a_3a_4b_1} p_1^{\alpha_1} p_2^{\beta_1} \eta_{\mu_1\mu_4} - \epsilon_{\mu_2\mu_3\alpha_1\beta_1} f_{a_1a_3b_1} f_{a_2a_4b_1} p_1^{\alpha_1} p_3^{\beta_1} \eta_{\mu_1\mu_4} - \epsilon_{\mu_2\mu_3\alpha_1\beta_1} f_{a_1}$$
$$+ \epsilon_{\mu_2\mu_3\alpha_1\beta_1} f_{a_1a_2b_1} f_{a_3a_4b_1} p_3^{\alpha_1} p_4^{\beta_1} \eta_{\mu_1\mu_4} - \epsilon_{\mu_1\mu_4\alpha_1\beta_1} f_{a_1a_2b_1} f_{a_3a_4b_1} p_1^{\alpha_1} p_2^{\beta_1} \eta_{\mu_2\mu_3} - \epsilon_{\mu_1\mu_4\alpha_1\beta_1} f_{a_1}$$
$$- \epsilon_{\mu_1\mu_4\alpha_1\beta_1} f_{a_1a_2b_1} f_{a_3a_4b_1} p_3^{\alpha_1} p_4^{\beta_1} \eta_{\mu_2\mu_3} + \epsilon_{\mu_1\mu_3\alpha_1\beta_1} f_{a_1a_2b_1} f_{a_3a_4b_1} p_1^{\alpha_1} p_2^{\beta_1} \eta_{\mu_2\mu_4} - \epsilon_{\mu_1\mu_3\alpha_1\beta_1} f_{a_1}$$
$$- \epsilon_{\mu_1\mu_3\alpha_1\beta_1} f_{a_1a_2b_1} f_{a_3a_4b_1} p_3^{\alpha_1} p_4^{\beta_1} \eta_{\mu_2\mu_4} + \epsilon_{\mu_1\mu_2\alpha_1\beta_1} f_{a_1a_3b_1} f_{a_2a_4b_1} p_1^{\alpha_1} p_3^{\beta_1} \eta_{\mu_3\mu_4} + \epsilon_{\mu_1\mu_2\alpha_1\beta_1} f_{a_1}$$
$$- \epsilon_{\mu_1\mu_2\alpha_1\beta_1} f_{a_1a_2b_1} f_{a_3a_4b_1} p_3^{\alpha_1} p_4^{\beta_1} \eta_{\mu_3\mu_4} - \epsilon_{\mu_1\mu_2\mu_3\mu_4} f_{a_1a_2b_1} f_{a_3a_4b_1} p_1 \cdot p_2 + \epsilon_{\mu_1\mu_2\mu_3\mu_4} f_{a_1a_3b_1} f_{a_2}$$
$$- \epsilon_{\mu_1\mu_2\mu_3\mu_4} f_{a_1a_4b_1} f_{a_2a_3b_1} p_1 \cdot p_4 - \epsilon_{\mu_1\mu_2\mu_3\mu_4} f_{a_1a_4b_1} f_{a_2a_3b_1} p_2 \cdot p_3 + \epsilon_{\mu_1\mu_2\mu_3\mu_4} f_{a_1a_3b_1} f_{a_2a_4b_1} p_2$$
$$- \epsilon_{\mu_1\mu_2\mu_3\mu_4} f_{a_1a_2b_1} f_{a_3a_4b_1} p_3 \cdot p_4 \Big)$$
$$+ 2iv^2 \bar{g}_s^2 \epsilon_{\mu_1\mu_2\mu_3\mu_4} C^{\phi\tilde{G}} \left(f_{a_1a_4b_1} f_{a_2a_3b_1} - f_{a_1a_3b_1} f_{a_2a_4b_1} + f_{a_1a_2b_1} f_{a_3a_4b_1}\right)$$