Radiative symmetry breaking from interacting UV fixed points

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Outline

- Motivation; the hierarchy versus triviality problem
- RG flows and the asymptotic safety idea
- Asymptotically safe 4D QFTs
- Coleman-Weinberg? (no)
- Adding relevant operators
- Radiative symmetry breaking
Motivation: two problems to do with scalars
Why is the Weak Scale so much lower than the Planck Scale - and how is it protected?

More precisely perturbation theory with a higgs scalar is suspect: very “massive states” dominate any perturbative calculation to do with higgs physics.

Actually don’t even need a heavy resonance: this can be true for some other rapid change (in e.g. beta functions) at a high scale.

**The hierarchy problem:**

Why is the Weak Scale so much lower than the Planck Scale - and how is it protected?

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**The hierarchy problem:**
The hierarchy problem:

Candidate symmetries:

- Higgs is a Goldstone mode of some broken global symmetry (like the pions in chiral symmetry breaking) with breaking scale of a few TeV.

- Supersymmetry - relates boson to fermions. Divergences cancel level by level. Phenomenology requires soft (a.k.a. dimensionful) breaking.

- Scaling symmetry - Higgs is the Goldstone mode of a broken scale invariance (a.k.a. dilaton) (a trivial perturbative example of this is the Standard Model with vanishing higgs mass, but it can occur in nonperturbative models based on AdS/CFT). (The subject here - but not Coleman-Weinberg!!)

- Misaligned Supersymmetry - even non-supersymmetric non-tachyonic strings are finite. (Alternative route to naturalness) (SAA+Dienes+Mavroudi)
The triviality problem:

Scalars lead to Landau poles:

=> the theory is UV incomplete

But trying to UV complete it result in the hierarchy problem again! (see previous comments)
QCD is (unlike SUSY) a UV complete theory. Why?

1. **There is no hierarchy problem:** quark masses are protected by chiral symmetry

2. **There is no triviality problem:** QCD is asymptotically free

Note the philosophy of QCD: we do not mind running masses because they do not upset the Gaussian UV fixed point. We simply measure them and let them run. Or to put it another way: they are “relevant” operators that are effectively zero in the UV. They do not need to run to zero in the UV! (We also don’t care too much about couplings blowing up in the IR.)
RG flows and the asymptotic safety idea
The Basic idea

Weinberg used this as a basis for his proposal of UV complete theories.
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Interacting UV fixed point $\Rightarrow$ finite anomalous dimensions

In a field theory replace $1/\epsilon$ with $1/\gamma$ $\Rightarrow$ divergences cured
The Basic idea

Weinberg used this as a basis for his proposal of UV complete theories

Gaussian IR fixed point $\Rightarrow$ perturbative

Interacting UV fixed point $\Rightarrow$ finite anomalous dimensions

In a field theory replace $1/\epsilon$ with $1/\gamma$ $\Rightarrow$ divergences cured
Categorise the content of a theory as follows

**Irrelevant operators:** would disrupt the fixed point - therefore asymptotically safe theories have to emanate precisely from UV fixed point where they are zero (exactly renormalizable trajectory)

**Marginal operators:** involved in determining the UV fixed point where they become *exactly* marginal

**Relevant operators:** become “irrelevant” in the UV but may determine the IR fixed point.

Note relevant operators still have “infinities” - just as quark masses, they still run at the FP just like any other relevant operator: but being relevant they do not affect the FP. (And by definition they become less important the higher you go in energy.)
Simple example of flow - normal QCD:

\[ \partial_t \alpha = -B \alpha^2 \quad t = \log \frac{\mu}{\mu_0} \]

This theory has *unstable* fixed point at \( \alpha = 0 \). Asymptotically free if \( B > 0 \)
Weinberg’s original set-up

If $A>0$, $B>0$, this theory has *unstable* UV fixed point at $\alpha = A/B$ and *stable* one at IR $\alpha = 0$

\[ \partial_t \alpha = A \alpha - B \alpha^2 \]

Gastmans et al’78
Weinberg ’79
Peskin ’80
Gawedski, Kupiainen ’85
Kawai et al’90
de Calan et al ‘91
Morris ’04
Turns out $C>0$, $B>0$: theory has stable IR fixed point at $\alpha = B/C$ and unstable one in UV $\alpha = 0$.

Take QCD with $SU(N_C)$ and $N_F$ fermions but very large numbers of particles …

$$\partial_t \alpha = -B\alpha^2 + C\alpha^3$$

$$B \propto \epsilon = \frac{N_F}{N_C} - \frac{11}{2}$$

Note perturbativity: $B \ll C$

requires many fields (Veneziano limit) with $N_F \approx 11N_C/2$

Familiar from Seiberg duality and weakly coupled $N_F \lesssim 3N_C$ $\mathcal{N} = 1$ supersymmetry
What about Asymptotic safety in 4D field theory?

Again would have …

\[ \partial_t \alpha = -B \alpha^2 + C \alpha^3 \]

But requires $C<0$, $B<0$, this theory has stable IR fixed point at $\alpha = 0$ and unstable UV one at $\alpha = B/C$

At $t \to \infty$ the coupling ends up here (and fields have finite anomalous dimensions)

Again perturbativity would require \[ N_F \approx \frac{11 N_C}{2} \]
Asymptotic safety in 4D QFT
That was a one coupling cartoon: real situation requires several couplings to realise

Litim & Sannino ’14

In order to get this behaviour need to add scalars and Yukawa couplings

\[ \mathcal{L} = -\frac{1}{2} \text{Tr} F_{\mu \nu} F^{\mu \nu} + \text{Tr} (\bar{Q} i \not{D} Q) + y \text{Tr} (\bar{Q} H Q) + \text{Tr} (\partial_{\mu} H^{\dagger} \partial^{\mu} H) - u \text{Tr} [(H^{\dagger} H)^2] - v (\text{Tr} [H^{\dagger} H])^2, \]

\( H \) is an \( N_F \times N_F \) scalar

Initially have \( U(N_F)_L \times U(N_F)_R \) flavour symmetry
The asymptotically safe theories of ref.[1] that we will be using here lie somewhere between these two extremes. By choosing a theory with a weakly interacting UV fixed point we recover the benefits of predictivity and control over the effective potential, but at the same time keep the theory under good perturbative control. This optimisation is reminiscent of the Banks-Zaks IR fixed point [49], which can be made arbitrarily weakly interacting and hence perturbatively tractable, in a particular (Veneziano) large-colour/large-flavour limit. Of course this work follows on from a large body of literature that has discussed asymptotic safety and more generally the consequences of UV scale invariance both with and without gravity: [48, 50–59]). (For a review see [60]). The object of this paper is to place radiative symmetry breaking in such frameworks on the same footing as it is in the MSSM.

II. THE THEORY, UV FIXED POINT AND CRITICAL CURVE

We begin by describing the behaviour of the weakly interacting gauge-Yukawa theories that we will be using, and in particular their phase diagrams and RG flow. Consider a theory with $SU(N_C)_{\text{gauge}}$ fields $A_a^\mu$ and field strength $F_a^\mu \nu$ ($a = 1, \cdots, N_C$), $N_F$ flavours of fermions $Q_i$ ($i = 1, \cdots, N_F$) in the fundamental representation, and a $N_F \times N_F$ complex matrix scalar field $H$ uncharged under the gauge group. At the fundamental level the Lagrangian is

$$L = L_{\text{YM}} + L_{\text{F}} + L_{\text{Y}} + L_{\text{H}} + L_{\text{U}} + L_{\text{V}},$$

with

$$L_{\text{YM}} = \frac{1}{2} \text{Tr} F_a^\mu \nabla F_a^\mu + \text{Tr} \frac{y}{D} Q_i H + y \text{Tr} Q_i H^{\dagger} Q_i H + \text{Tr} \left( \frac{\partial}{\partial \mu} H^{\dagger} \partial \mu H \right).$$

(1)

where $\text{Tr}$ indicates the trace over both color and flavor indices. The model has four coupling constants given by the gauge coupling, the Yukawa coupling $y$, and the quartic scalar couplings $u$ and the double-trace scalar coupling $v$:

$$(\alpha_g = g^2 N_C / (4\pi)^2, \quad \alpha_y = y^2 N_C / (4\pi)^2)$$

(2)

We have already re-scaled the coupling constants by the appropriate powers of $N_C$ and $N_F$ to work in the Veneziano limit. When necessary we will use a shorthand notation with $i = (g, y, h, v)$. As mentioned in the Introduction we will be considering the large colour and large flavour Veneziano limit, in order to have an interacting fixed point which is nevertheless arbitrarily weakly coupled. Therefore it is convenient to introduce a control parameter which in the Veneziano limit is a continuous and arbitrarily small constant $\epsilon = N_F / N_C^{11/2}$.

(3)

Asymptotic freedom is lost for positive values of $\epsilon$.

Ref.[1] discovered a number of fixed points for this model. However there is one fixed point that is unique in that it has only one relevant direction with the other three being irrelevant. Since every relevant direction loses predictivity (as it is formally zero at the fixed

$$\beta_g = \alpha_g^2 \left[ \frac{4}{3} \epsilon + \left( 25 + \frac{26}{3} \epsilon \right) \alpha_g - 2 \left( \frac{11}{2} + \epsilon \right)^2 \alpha_y \right]$$

$$\beta_y = \alpha_y \left[ (13 + 2\epsilon) \alpha_y - 6 \alpha_g \right]$$

These equations describe the RG flow of the coupling constants in the parameter space $(\alpha_g, \alpha_y)$.
Four couplings - flow could in principle be four dimensional

\[
\alpha_g = \frac{g^2 N_C}{(4\pi)^2}, \quad \alpha_y = \frac{y^2 N_C}{(4\pi)^2}, \quad \alpha_h = \frac{u N_F}{(4\pi)^2}, \quad \alpha_v = \frac{v N_F^2}{(4\pi)^2}
\]

but …
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but …

UV fixed point

1D exactly renormalisable trajectory!

Gaussian IR fixed point
Along the separatrix/critical-curve/exact-trajectory can parameterise the flow in terms of \( \alpha_g(t) \)

\[
\alpha_y(t) = \frac{6}{13} \alpha_g(t) , \\
\alpha_h(t) = \frac{3 \sqrt{23} - 1}{26} \alpha_g(t) , \\
\alpha_v(t) = \frac{3 \sqrt{20 + 6 \sqrt{23}} - 6 \sqrt{23}}{26} \alpha_g(t) ,
\]

At the fixed point it is arbitrarily weakly coupled, \( \alpha_g^* = 0.4561 \epsilon \), where \( \epsilon = \frac{N_F}{N_C} - \frac{11}{2} \)
Coleman Weinberg?
(no)
Recap of the idea

- The SM is “classically” scale invariant - tree level Lagrangian has no mass
- Coleman Weinberg mechanism leads to spontaneous breaking at a scale because the scale invariance is anomalous. (Huge amount of interest since 2012)
- Compute effective potential and renormalize it

\[ V_{\text{eff}} = \frac{\lambda}{4!} |\phi|^4 + \frac{3g^4}{64\pi^2} |\phi|^4 \left( \log \frac{|\phi|}{\mu} - \frac{25}{6} \right) \]

\[ \frac{\partial^2 V}{\partial \phi^2} |_{\phi=0} = 0 \quad \frac{\partial^4 V}{\partial \phi^4} |_{\phi=\mu} = \lambda \]

We imposed by hand no generation of mass terms!
Minimization leads to dimensional transmutation

\[ \langle \phi \rangle = \mu e^{\frac{11}{6} - \frac{4\pi^2\lambda}{9g^4}} \]
Hence we define the effective space and assume that a negative mass term, for example choose the real trace direction …

- **Heuristically unlikely to work from a UV fixed point:** CW is all about IR scale invariance where $\phi=0$ - which is why it is a strange starting point for solving the problems of large UV thresholds.

- Proof (already shown numerically by Litim, Mojaza, Sannino but can do it analytically): for example choose the real trace direction …

\[
H = \frac{\phi}{\sqrt{2N_F}} \mathbb{1}_{N_F \times N_F} \quad \rightarrow \quad V_{\text{class}}^{(4)} = \frac{4\pi^2}{N_F^2} (\alpha_h + \alpha_v) \phi^4
\]
- Effectively \[ \lambda = 32 \pi^2 \frac{3}{N_F^2} (\alpha_h + \alpha_v) \]

- Also define \[ \kappa = 32 \pi^2 \frac{1}{N_F^2} (3\alpha_h +\alpha_v) \]

\[
V = \frac{\lambda}{4!} \phi^4 + \frac{m_\phi^2}{2} \phi^2 + \frac{1}{64\pi^2} \left( m_\phi^2 + \frac{\lambda}{2} \phi^2 \right)^2 \left( \log \frac{m_\phi^2 + \frac{\lambda}{2} \phi^2}{\mu^2} - \frac{3}{2} \right) \\
- \frac{(4\pi)^2}{4N_FN_C} \alpha_y^2 \phi^4 \left( \log \frac{(4\pi)^2 \alpha_y \phi^2}{\sqrt{N_FN_C} \mu^2} - \frac{3}{2} \right) \\
+ \frac{(N_F^2 - 1)}{64\pi^2} \left( \frac{\kappa}{2} \phi^2 \right)^2 \left( \log \frac{\kappa}{2} \phi^2 \mu^2 - \frac{3}{2} \right) + \frac{N_F^2}{64\pi^2} \left( \frac{\lambda}{6} \phi^2 \right)^2 \left( \log \frac{\lambda}{6} \phi^2 \mu^2 - \frac{3}{2} \right)
\]
Effectively \[ \lambda = 32\pi^2 \frac{3}{N_F^2} (\alpha_h + \alpha_v) \]

Also define \[ \kappa = 32\pi^2 \frac{1}{N_F^2} (3\alpha_h + \alpha_v) \]

Corrections all of order \( \alpha \lambda \), so no perturbative minimum without a mass-squared for \( \phi \)

\[
V = \frac{\lambda}{4!} \phi^4 + \frac{m_\phi^2}{2} \phi^2 + \frac{1}{64\pi^2} \left( m_\phi^2 + \frac{\lambda}{2} \phi^2 \right)^2 \left( \log \frac{m_\phi^2 + \frac{\lambda}{2} \phi^2}{\mu^2} - \frac{3}{2} \right) \\
- \frac{(4\pi)^2}{4N_F N_C} \alpha_y^2 \phi^4 \left( \log \frac{(4\pi)^2 \alpha_y \phi^2}{\sqrt{N_F N_C} \mu^2} - \frac{3}{2} \right) \\
+ \frac{(N_F^2 - 1)}{64\pi^2} \left( \frac{\kappa}{2} \phi^2 \right)^2 \left( \log \frac{\kappa \phi^2}{\mu^2} - \frac{3}{2} \right) + \frac{N_F^2}{64\pi^2} \left( \frac{\lambda}{6} \phi^2 \right)^2 \left( \log \frac{\lambda}{6} \phi^2}{\mu^2} - \frac{3}{2} \right)
\]
Adding relevant operators mass-squareds
Solve Callan Symanzik eqn for them as usual =>

- **warm-up**: first restrict ourselves to the diagonal direction where mass-squared term looks like the following operator:

\[ V \supset \frac{m_\phi^2}{4NF} \left( \text{Tr}(H + H^\dagger) \right)^2 \]

\[ \bar{\beta} = \frac{d\lambda^{(n)}(t)}{dt} = \frac{\partial \lambda_{\text{eff}}^{(n)}}{\partial t} + n \bar{\gamma} \lambda^{(n)} \]

Anomalous dimension of fields

t-dependence in one-loop calculation of V
Solve Callan Symanzik eqn for them as usual =>

For mass-squareds, by dimensions have contributions from cross-terms only …

\[ V \supset \frac{m_\phi^2}{2} \phi^2 \left( 1 + \frac{\lambda t}{16\pi^2} \right) \]

Using the solutions along the separatrix:

\[ \beta_m^2 = m_\phi^2 \left( \frac{\lambda}{16\pi^2} + 2\gamma \right), \]

\[ = 2\alpha_y + \frac{6}{N_F^2} (\alpha_v + \alpha_h) \]

\[ = f \alpha_g \quad \left( f = \frac{12}{13} \left[ 1 + \frac{3}{4N_F^2} \left( \sqrt{20 + 6\sqrt{23}} - 1 - \sqrt{23} \right) \right] \right) \]

i.e. mass-squared scales with the gauge coupling like all the marginal couplings …
We find \textit{multiplicative} renormalisation …

\[
m_{\phi}(t) = m_0 \left( \frac{\alpha_g}{\alpha_g(0)} - 1 \right) \left( \frac{\alpha_g}{\alpha_g(0)} - 1 \right)^{-\frac{3f}{4\epsilon}}
\]

\[
\alpha_g = 0.4561 \epsilon
\]

In principle … \( m_*^2 = m_0^2 \left( \frac{\alpha_g}{\alpha_g(0)} - 1 \right) \left( \frac{\alpha_g}{\alpha_g(0)} - 1 \right)^{\frac{3f}{4\epsilon}} \) but you should just think of it as an RG invariant that defines this particular trajectory. (Every relevant operator will have an associated invariant.) It has the same status as the chiral quark masses.
Radiative symmetry breaking
Criticism of the simplest example...

- **Purely** multiplicative: Hence the mass-squared has to be negative along the whole trajectory.

- **We cheated**: in the sense that we ignored all the orthogonal directions!! These also get contributions at one-loop even though their masses were zero at tree-level.

In order to address both these, organise the discussion in terms of the $U(N_F) \times U(N_F)$ flavour symmetry that we break with the mass-squareds:

$$H = \frac{(h_0 + ip_0)}{\sqrt{2N_F}} \mathbb{1}_{N_F \times N_F} + (h_a + ip_a)T_a$$
Non-trivial simple example...

Seek to add a set of mass-squared operators whose flavour structure is isomorphic under RG: simple example

\[
V_{\text{class}}^{(2)} = m_0^2 \text{Tr}(H^\dagger H) + 2\Delta^2 \sum_a \text{Tr}(T_a H^\dagger)\text{Tr}(T_a H)
\]

\[
\begin{align*}
\Delta^2 & \quad m_0^2 \\
& \quad m^2_h = m_\alpha^2 = m_0^2 + \Delta^2 \\
& \quad m_0^2 = m_\alpha^2 = m_0^2 \\
\end{align*}
\]
Following the same procedure and after some work find the following answer in terms of two RG invariants (one for each independent bit of the flavour structure) (where \( \nu = (1 - 1/NF^2) \)):

\[
m_0^2 = \tilde{m}_*^2 \left( \frac{\alpha_g^*}{\alpha_g} - 1 \right)^{-\frac{3f m_0}{4e}} - \Delta^2 \nu \left( \frac{\alpha_g^*}{\alpha_g} - 1 \right)^{-\frac{3f \Delta}{4e}},
\]

\[
m_{a=1...N_F-1}^2 = \tilde{m}_*^2 \left( \frac{\alpha_g^*}{\alpha_g} - 1 \right)^{-\frac{3f m_0}{4e}} + \Delta^2 (1 - \nu) \left( \frac{\alpha_g^*}{\alpha_g} - 1 \right)^{-\frac{3f \Delta}{4e}}
\]

\[f m_0 > f \Delta\]

Dies away quickly \textit{in the IR} \quad Dies away slowly \textit{in the IR}
Starting values get *relatively* closer in UV (note the masses are all growing *in absolute* terms in the UV) - full flavour symmetry restored precisely at fixed point.

\[ \Delta^2_{(0)} \{ m^2 \} \quad \{ m^2_{0(0)} \} \]

The sum of the mass-squareds quickly dies to zero in IR:

\[ \sim \Delta^2_*/N_F^2 \quad \sim \Delta^2_* \]
Figure 2: A mass-squared that is smaller than the average by 5% being driven negative radiatively, (where the initial value at \( t = 0 \) is 0.99). We take \( \tilde{c} = 0 \) in the Veneziano limit (\( N_F \sim 1 \)).

One has

\[
\bigg( g_{\text{min}} \bigg) \sim \frac{1}{2} g^*.
\]

In other words the minimum forms at precisely the scale where the theory is passing from the UV fixed point, and the flow is coming under the more standard influence of the Gaussian IR fixed point. Finally note that if we had chosen negative \( m^2 \) the reversed pattern of breaking would have occurred, with the trace \( h_0 \) direction being the only stable and very heavy direction, with a mass-squared balancing order \( N^2 \) very small negative mass-squareds for all the orthogonal directions.

IV. CONCLUSIONS

We have studied the stability properties of the class of perturbative UV fixed point theories introduced in ref.[1], in the presence of additional scalar mass-squared terms. It is important to realise that such terms, being relevant operators, may take any value in a scenario of asymptotic safety without disrupting the fixed point. As such their status is similar to that of the quark masses in QCD: they are simply set by hand at some scale and are fully controlled and multiplicatively renormalised along the entire RG trajectory. Indeed the value of all the relevant operators everywhere along the flow is completely determined by a set of corresponding RG invariants.

This general picture, in which the trajectories of relevant operators (for example \( m^2 \star \) in our case) are determined by a set of tunable RG invariants that defines a particular model, while the marginal operators are all (except for one) determined by a UV fixed point, is a familiar one in the context of the exact renormalisation group. However it is certainly novel to be able to treat it perturbatively.

Such a treatment reveals that these theories exhibit an interesting form of calculable radiatively induced symmetry breaking, that is exactly analogous to that in the MSSM[6]. It was found that a generic set of positive but flavour violating mass-squared terms

\[
\begin{align*}
\alpha_{g\text{,min}} & \rightarrow 0 \\
\frac{1}{2} \alpha^* \\
\frac{1}{2} g^* \\
\end{align*}
\]

\[
\begin{align*}
\alpha_{g\text{,min}} & \rightarrow 0 \\
\frac{1}{2} \alpha^* \\
\frac{1}{2} g^* \\
\end{align*}
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\frac{1}{2} \alpha^* \\
\frac{1}{2} g^* \\
\end{align*}
\]
Summary

- Considered perturbative asymptotically safe QFTs (gauge-Yukawa theories that require scalars)
- UV fixed points do not prefer any mass-squared - they are relevant operators so simply take any value described by an RG-invariant (multiplicative renormalisation)
- Deviation from zero = breaking of scale invariance, c.f. non-zero quark masses = breaking of chiral symmetry
- Positive mass-squareds can be driven negative in the IR, akin to radiative symmetry breaking in MSSM
- Minimum generated radiatively
- The effect depends on the explicit breaking of flavour structure in the RG invariants.