1 Introduction

• SUSY hybrid inflation is a very promising inflation scenario.
• It predicts $n_s$ in the observed range of 0.96-0.97.
• SUGRA corrections are under control for inflaton values $\ll m_P$.
• Here, we merge SUSY hybrid inflation with axion physics.
• The intermediate PQ scale $f_a$ can generate the MSSM $\mu$ term.
• Namely, the 'bare' $\mu$ term is forbidden through symmetries, but it arises from a higher dimensional superpotential term.
• The resulting $\mu \sim f_a^2/m_P \sim \text{TeV}$, the desired magnitude.
• The tensor-to-scalar ratio $r$ is negligible in this simple model.
• Both the axion and the LSP would contribute to dark matter.
• We investigate how the PQ transition proceeds in a SUSY model with hybrid inflation in accord with observations.
• A potential problem is the generation of unacceptably large axion isocurvature perturbations.
• To avoid them the PQ field during inflation must be fairly large.
• This is achieved by higher order terms in the Kähler potential.
• They produce a negative $\text{mass}^2$ for the PQ field thereby shifting its vacuum value to become large during inflation.
• Thus, the PQ field is large during inflation and inflaton oscillations.
• It then gradually drifts to the desired low energy PQ vacuum.
• So, the PQ symmetry is spontaneously broken already during inflation and remains so thereafter.
• The axion domain wall problem, which appears if the spontaneous breaking of $U(1)_{PQ}$ occurs after inflation, does not arise.
• Our numerical study shows how the PQ field develops during inflation and inflaton oscillations until low energies.
• The system contains not only the saxion and the axion but also an extra complex Higgs field and a four component Dirac axino.
• We estimate the amplitude of the PQ oscillations when soft SUSY breaking takes over and calculate the ensuing abundances.
• The decay time and thermal density of the PQ states is estimated.

2 Model and the PQ transition

• Consider a SUSY model based on the left-right symmetric gauge group $G_{LR} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.
• The superfields are the gauge singlets $S, N, \bar{N}$, the Higgses $H = (1, 1, 2)_1, \bar{H} = (1, 1, 2)_{-1}$ causing $G_{LR} \to G_{SM}$, the electroweak Higgs $h = (1, 2, 2)_0$, and the matter superfields $q_i, q^c_i, l_i, l^c_i$.
• The model has a R symmetry $U(1)_R$ with the superfields charges
  \[
  R : S(2), N(0), \bar{N}(2), H, \bar{H}(0), h(1), q_i, l_i(0), q^c_i, l^c_i(1).
  \]
• Also, a PQ symmetry $U(1)_{PQ}$ with the superfields charges
  \[
  PQ : S(0), N(-1), \bar{N}(3), H, \bar{H}(0), h(1), q_i, l_i(-1), q^c_i, l^c_i(0).
  \]
• The symmetries allow $\bar{H}H^c_i l^c_j$ for right-handed neutrino masses.
• The ‘bare’ $\mu$ term $h^2$ is forbidden, but the term $N^2 h^2$ is allowed and generates a $\mu$ term of the desired magnitude.
• The soft SUSY breaking terms break $U(1)_R$ explicitly to its $Z_2$ subgroup under which $h, q^c_i, l^c_i$ are odd.
• This $Z_2$, combined with the $Z_2$ center of $SU(2)_L$ provides the well-known $Z_{mp}^2$ under which the matter fields are odd.
• So, the apparent spontaneous breaking of the $Z_2$ subgroup of $U(1)_R$ does not lead to domain walls.
• This $Z_2$ is replaced by the equivalent $Z_{mp}^2$, which is unbroken.
• $U(1)_{PQ}$ is broken explicitly by instantons to $Z_6$, which breaks spontaneously during inflation and the walls are inflated away.
• The symmetries imply perturbative baryon number conservation.
• The superpotential relevant for inflation and the PQ system is

$$W = \kappa S \left( M^2 - H \bar{H} \right) + \frac{\lambda}{m_P} N^3 \bar{N}.$$

• The Kähler potential is taken to be

$$K = |S|^2 + |H|^2 + |\bar{H}|^2 + |N|^2 + |\bar{N}|^2 + \frac{\alpha}{4m_P^2} |S|^4 + \left( c_1 |S|^2 + c_2 |H|^2 + c_3 |\bar{H}|^2 + c_4 \left( H \bar{H} + h.c. \right) \right) \frac{|N|^2}{m_P^2} + \left( c_5 |S|^2 + c_6 |H|^2 + c_7 |\bar{H}|^2 + c_8 \left( H \bar{H} + h.c. \right) \right) \frac{|\bar{N}|^2}{m_P^2}.$$

• $M$ is a superheavy mass and $\kappa, \lambda, \alpha, c_i$ are dimensionless constants taken real with $\alpha > 0, c_1 > 1, c_2, c_3 \geq 1, c_5, c_6, c_7 < 1$. 
Expand the F-term potential in powers of $m_{P}^{-1}$ up to 2nd order:

$$V = \kappa^2 |M^2 - H\bar{H}|^2 (1 + |H|^2 + |\bar{H}|^2 - \alpha |S|^2 - (c_1 - 1)|N|^2 + (1 - c_5)|\bar{N}|^2) + \kappa^2 |S|^2 (|H|^2 + |\bar{H}|^2) (1 + |S|^2 + |H|^2 + |\bar{H}|^2) + 4\kappa^2 |S|^2 |H|^2 |\bar{H}|^2 - 2\kappa^2 M^2 |S|^2 (H\bar{H} + h.c.) - \kappa^2 |S|^2 ((c_3 - 1)|H|^2 + (c_2 - 1)|\bar{H}|^2) |N|^2 + \kappa^2 |S|^2 ((1 - c_5)|H|^2 + (1 - c_6)|\bar{H}|^2) |\bar{N}|^2 + \lambda^2 |N|^4 (|N|^2 + 9|\bar{N}|^2) + m_{3/2}^2 (\varepsilon_1 |N|^2 + \varepsilon_2 |\bar{N}|^2) + m_{3/2} (\lambda A N^3 \bar{N} + h.c.).$$

- We included here only the soft terms for $N$, $\bar{N}$ and put $m_P = 1$. We set $A = 0$ and take $\varepsilon_2 > 0$.
- For $c_5$, $c_6$, $c_7 < 1$, the minimum of $V$ is always at $\bar{N} = 0$ and, thus, we safely set $\bar{N} = 0$ throughout the discussion.
- To achieve spontaneous breaking of the PQ symmetry, we choose $\varepsilon_1 < 0$ and absorb it redefining $m_{3/2}^2$.
- Take the D-flat direction $H = \bar{H}^*$ containing the SUSY vacuum.
- By $U(1)_R$, $U(1)_{PQ}$, $U(1)_{B-L}$ rotations, we make $S$, $N$, $H$, $\bar{H}$ real and define almost canonically normalized real scalars $\sigma$, $\chi$, $h$:

$$S = \frac{\sigma}{\sqrt{2}}, \quad N = \frac{\chi}{\sqrt{2}}, \quad H = \bar{H} = \frac{h}{2}.$$

- In terms of $\sigma$, $\chi$, $h$, the potential is

$$V = \kappa^2 \left(M^2 - \frac{h^2}{4}\right)^2 \left(1 + \frac{h^2}{2} - \alpha \frac{\sigma^2}{2} - d_1 \frac{\chi^2}{2}\right) + \frac{\lambda^2}{8} \chi^6
\frac{\kappa^2}{4} \sigma^2 h^2 \left(1 + \frac{\sigma^2}{2} + h^2 - 2M^2 - d_2 \frac{\chi^2}{2}\right) - \frac{1}{2} m_{3/2}^2 \lambda^2,$$
where $d_1 \equiv c_1 - 1 > 0$, $d_2 \equiv (c_2 + c_3)/2 - 1 \geq 0$.

- The $\chi$-dependent part of $V$ may be written as
  \[ V_\chi = -\frac{1}{2} m_\chi^2 \chi^2 + \frac{\chi^2}{8} \chi^6, \]
  where
  \[ m_\chi^2 = d_1 \kappa^2 \left( M^2 - \frac{h^2}{4} \right)^2 + d_2 \frac{\kappa^2}{4} \sigma^2 h^2 + m_{3/2}^2. \]

- $V_\chi$ is minimized at $\chi = \chi_m$:
  \[ |\chi_m| = \sqrt{2} \left( \frac{m_\chi^2}{3 \lambda^2} \right)^{\frac{1}{4}}. \]

- $c_1 > 1$, $c_2$, $c_3 \geq 1 \Rightarrow m_\chi^2 > 0$ and the minimum during inflation and inflaton oscillations is shifted away from $\chi = 0$.

- This is crucial for controlling the axion isocurvature perturbations and avoiding the axion domain wall problem.

- For $\sigma > \sigma_c \simeq \sqrt{2} M$, the hybrid inflation path at $h = 0$ is stable.

- The inflationary potential is
  \[ V \overset{h=0}{=} \kappa^2 M^4 \left( 1 - \alpha \frac{\sigma^2}{2} \right) + V_\chi \simeq \kappa^2 M^4, \quad \text{and} \quad m_\chi^2 \simeq d_1 \kappa^2 M^4. \]

- So, $|\chi_m|$ during inflation becomes
  \[ |\chi_{\text{inf}}| \simeq \sqrt{2} \left( \frac{d_1 \kappa^2 M^4}{3 \lambda^2} \right)^{\frac{1}{4}}. \]
• Since $m = H_{\text{inf}}$, $\chi$ is expected to quickly reach the above value.

• To the inflationary potential we add the radiative corrections

$$V_{\text{rad}} = \kappa^2 M^4 \left( \frac{\delta_h}{2} \right) \ln \frac{\sigma^2}{\sigma_c^2}, \quad \delta_h = N_h \frac{\kappa^2}{8\pi^2}$$

where $N_h = 2$ is the dimensionality of $H, \bar{H}$.

• The potential during inflation is then

$$V_{\text{inf}} \simeq \kappa^2 M^4 \left( 1 + \frac{\delta_h}{2} \ln \frac{\sigma^2}{\sigma_c^2} - \frac{\alpha \sigma^2}{2} \right) \simeq \kappa^2 M^4.$$ 

• The slow-roll parameters are

$$\epsilon \equiv \frac{1}{2} \left( \frac{V'_{\text{inf}}}{V_{\text{inf}}} \right)^2 \simeq \frac{1}{2} \alpha \delta_h \frac{(1 - x)^2}{x}, \quad \eta \equiv \frac{V''_{\text{inf}}}{V_{\text{inf}}} \simeq -\alpha \frac{1 + x}{x},$$

$$\xi \equiv \left( \frac{V'_{\text{inf}}}{V_{\text{inf}}} \right) \left( \frac{V'''_{\text{inf}}}{V_{\text{inf}}} \right) \simeq 2\alpha^2 \frac{1 - x}{x^2}, \quad \text{with} \quad x \equiv \frac{\alpha \sigma^2}{\delta_h} < 1.$$ 

• Inflation ends when $\sigma$ reaches the value $\sigma_{\text{end}}$ with

$$\sigma_{\text{end}}^2 \simeq \max\{2M^2, \delta_h\},$$

depending on whether inflation ends with a waterfall or smoothly.

• The number $N(x_{\text{in}})$ of e-foldings between $\sigma = \sigma_{\text{in}}$ and $\sigma = \sigma_{\text{end}}$ corresponding to $x_{\text{in}}$ and $x_{\text{end}}$ is

$$N(x_{\text{in}}) \simeq \frac{1}{2\alpha} \ln \frac{1 - x_{\text{end}}}{1 - x_{\text{in}}}.$$ 

• So, if $x = x_*$ at horizon exit of the pivot scale $k_* = 0.05 \text{ Mpc}^{-1}$,

$$N_* \equiv N(x_*) \simeq \frac{1}{2\alpha} \ln \frac{1}{1 - x_*}.$$
The scalar spectral index $n_s$ is

$$n_s \simeq 1 + 2\eta_* \simeq 1 - 2\alpha \frac{1 + x_*}{x_*} \simeq 1 - \frac{1}{N_*} \left( \frac{1 + x_*}{x_*} \ln \frac{1}{1 - x_*} \right).$$

Its running $\alpha_s \equiv dn_s/d\ln k$ and is the inflationary $V$ are

$$\alpha_s \simeq -2\xi_* \simeq -4\alpha^2 \frac{1 - x_*}{x_*^2}, \quad V = 24\pi^2 \epsilon_* A_s,$$

$A_s$ is the scalar power spectrum amplitude at horizon exit of $k_*$. Many SUSY inflation scenarios predict $n_s \simeq 1 - 1/N_* \simeq 0.98$.

A modified relation $n_s \simeq 1 - 2/N_*$, though, is in better agreement with observations and is easily obtain in our model for $x_* \simeq 0.5$.

Indeed, $x_* = 0.5$ and $N_* = 52 \Rightarrow \alpha \simeq 1/150, n_s \simeq 0.96, \alpha_s \simeq -3.56 \times 10^{-4}, M \simeq 2.17 \times 10^{-3}$ for $A_s = 2.215 \times 10^{-9}$.

We now put $\kappa = 0.01 \Rightarrow |\sigma_*| \simeq 0.01378, x_{\text{end}} \simeq 0.0248, \epsilon_* \simeq 4.22 \times 10^{-9}$, and the tensor-to-scalar ratio $r = 16\epsilon_* \simeq 6.76 \times 10^{-8}$.

So gravity waves are not observable.

$V$ during inflaton oscillations $\simeq E/2$ with $E$ being the energy density of the rapidly oscillating massive fields $\sigma$ and $h$.

So the position of the minimum of $V$ is

$$|\chi_m| \simeq \sqrt{2} \left( \frac{d E/2 + m_{3/2}^2}{3\lambda^2} \right)^{\frac{1}{4}} \text{ with } d = d_1 = d_2$$

As $d E/2$ approaches $m_{3/2}^2, |\chi_m|$ reaches smoothly the value

$$|\langle \chi \rangle| = \sqrt{2}|\langle N \rangle| \equiv f_a = \sqrt{2} \left( \frac{m_{3/2}^2}{3\lambda^2} \right)^{\frac{1}{4}},$$

which determines the PQ breaking scale $f_a$ (axion decay constant).
To confirm our understanding for the postinflationary evolution of $\chi$, we solve numerically the DEs for the dynamics of the system.

- We take $\kappa = 0.01$, $\lambda = 1$, $\alpha = 1/150$, $M = 2.17 \times 10^{-3}$ and the initial conditions $\sigma = 0.015$, $\dot{\sigma} = -1.86 \times 10^{-12}$, $h = 10^{-8}$, $\dot{h} = 0$, $\chi = 10^{-5}$, $\dot{\chi} = 0$.

- The initial $\dot{\sigma}$ is the velocity of $\sigma$ during inflation found numerically.

- We followed the evolution for 77 e-foldings out of which about 66.7 correspond to inflationary expansion.

- As an example, we show $\chi$ and $|\chi_m|$ as functions of $N_{\text{exp}}$ only for the period after inflation for $m_{3/2} = 10^{-13}$, $d_1 = d_2 = 1$.

- $N_{\text{exp}}$ is from $t = 0$ where the initial conditions were imposed.
• We see that \( \chi \) oscillates around the moving minimum \( \chi_m \).

• After 76 e-foldings, \( \chi \) reaches the low-energy PQ minimum and the amplitude of oscillations is severely reduced.

• We studied the evolution of \( \chi \) for various choices of \( m_{3/2}, d_1, d_2 \) including \( d_2 \leq 0 \). The emerging picture is basically the same.

• So our numerical findings support our theoretical expectations.

3 Relic density of the PQ fields

• From \( \delta W_1 = \lambda N^3 \bar{N} \), we obtain the potential in global SUSY:
  \[
  V = \lambda^2 |N|^4 (|N|^2 + 9 |\bar{N}|^2) - m_N^2 |N|^2 + m_{\bar{N}}^2 |\bar{N}|^2.
  \]
  Here \(-m_N^2 = \varepsilon_1 m_{3/2}^2\), \( m_{\bar{N}}^2 = \varepsilon_2 m_{3/2}^2 \) are the soft masses squared of \( N, \bar{N} \) and the soft A term is set to zero.

• \( V \) has a local maximum at \(|N| = |\bar{N}| = 0\) and the absolute minimum at
  \[
  |N| = \left( \frac{m_{\bar{N}}^2}{3\lambda^2} \right)^{\frac{1}{4}} \equiv \langle N \rangle, \quad |\bar{N}| = 0,
  \]
  corresponding to the low-energy PQ vacuum.

• Writing \( N = \langle N \rangle + \delta N \) with \( \delta N = (\delta \chi + ia)/\sqrt{2} \) in \( V \), we obtain a saxion \( \delta \chi \) with \( m_{\delta \chi}^2 = 4m_N^2 \), a massless axion \( a \), and an extra complex scalar \( \bar{N} = (\bar{\chi} + ia)/\sqrt{2} \) with \( m_{\bar{\chi}}^2 = 3m_N^2 + m_{\bar{N}}^2 \).

• Saxion decays to MSSM Higgsinos \( \tilde{h}_u, \tilde{h}_d \) via the Yukawa coupling
  \[
  -2\sqrt{2} \frac{\mu}{f_a} \delta N \tilde{h}_u \tilde{d}_d + h.c.,
  \]
  from \( \delta W_2 = \frac{1}{2} \lambda_\mu N^2 h^2 \), which also gives the \( \mu \) term with \( \mu = \lambda_\mu \langle N \rangle^2 \) after the PQ breaking.
• The saxion decay width is estimated to be
\[ \Gamma_d \simeq \frac{1}{\pi} \left( \frac{\mu}{f_a} \right)^2 m_{\delta \chi}. \]

• The energy density \( \rho_{\delta \chi} \) of coherent saxion oscillations behaves like pressureless matter and, thus, when saxion decays at \( t_d = \Gamma_d^{-1} \), is
\[ \rho_{\delta \chi}^d = \rho_{\delta \chi}^r \left( \frac{t_r}{t_d} \right)^\frac{3}{2}, \]
where \( \rho_{\delta \chi}^r \) is the energy density at the time \( t_r \) of reheating.

• The initial amplitude of saxion oscillations at \( t_{\text{osc}} \sim m_{\delta \chi}^{-1} \) is \( \sim f_a \).

• So, the initial energy density is
\[ \rho_{\delta \chi}^{\text{osc}} \sim \frac{1}{2} m_{\delta \chi}^2 f_a^2 \implies \rho_{\delta \chi}^r \sim \frac{1}{2} m_{\delta \chi}^2 f_a^2 \left( \frac{t_{\text{osc}}}{t_r} \right)^2, \]
since the universe is matter dominated between \( t_{\text{osc}} \) and \( t_r \).

• We then find that
\[ \rho_{\delta \chi}^d \sim \frac{1}{2\pi^2} m_{\delta \chi}^2 f_a^2 \left( \frac{t_d}{t_r} \right)^\frac{1}{2} \left( \frac{\mu}{f_a} \right)^4. \]

• For \( f_a = 10^{11} \text{ GeV} \) and \( \mu = m_{\delta \chi} = 1 \text{ TeV} \), we find \( t_d \simeq 3.14 \times 10^{13} \text{ GeV}^{-1} \) and, for \( T_r = 10^9 \text{ GeV} \), we get \( t_r \simeq 0.24 \text{ GeV}^{-1} \).

• We then obtain
\[ \rho_{\delta \chi}^d \simeq 58 \text{ GeV}^4 \ll \rho_{\text{rad}}^d \simeq 4.5 \times 10^9 \text{ GeV}^4, \]
and the energy density of saxion oscillations never dominates.

• We also find \( T_d \simeq 110 \text{ GeV} \) > the neutralino freeze-out temperature \( T_i \simeq m_{\text{LSP}}/25 \) if the neutralino mass \( m_{\text{LSP}} \lesssim 1.65 \text{ TeV} \).
• So, in this case, the neutralino relic abundance is not affected.

• Thermal saxion production is dominated by processes involving again the saxion-Higgsino-Higgsino coupling.

• The thermal saxion abundance $Y_{\delta\chi} \equiv \frac{n_{\delta\chi}}{s}$ is similar to that of $\tilde{a}$

$$Y_{\tilde{a}} \approx 10^{-5} \left( \frac{f_{\tilde{a}}}{10^{11} \text{ GeV}} \right)^{-2} \left( \frac{\mu}{10^3 \text{ GeV}} \right)^2 \approx 10^{-5}.$$  

• Thus, the thermal saxion energy density $\rho^d_{\delta\chi}$ at saxion decay is

$$\frac{\rho^d_{\delta\chi}}{\rho^d_{\text{rad}}} \approx \frac{4 Y_{\delta\chi} m_{\delta\chi}}{3 T_d} \approx 10^{-4} \ll 1$$

and the thermal saxions also remain always subdominant.

• The oscillations of complex Higgs $\tilde{N}$ are, as shown numerically, of vanishing amplitude and do not contribute to the energy density.

• Thermal production of $\tilde{N}$’s involves the coupling

$$2\sqrt{2} \frac{\mu}{f_{\tilde{a}}} \frac{m_{\tilde{a}}}{m_{\tilde{\chi}}} m_{\tilde{\chi}} \tilde{N} h_u h_d + h.c.,$$

where $m_{\tilde{a}} = \sqrt{3}m_N$ is the axino mass.

• $\tilde{N}$ decays into a pair of ordinary Higgses via the same coupling.

• So, $\tilde{N}$ also decays well before the freeze out of neutralinos remaining subdominant.

• The superpotential term $\delta W_1$ gives rise to the Yukawa coupling

$$-3\lambda \langle N \rangle^2 \tilde{N} \tilde{N} + h.c.$$  

yielding a four component axino $\tilde{a}$ with Dirac mass $m_{\tilde{a}} = \sqrt{3}m_N$.

• The axino decays into an ordinary Higgs and a Higgsino, again well before the neutralino freeze out and remains subdominant.
• So, saxion, $\tilde{N}$, $\tilde{a}$ do not affect the universe in any essential way.
• Thermal axions contribution to the effective number of $\nu$ is tiny.

4 Axion Isocurvature Perturbations

• The axion during inflation is massless and thus acquires a perturbation which, at horizon exit of the pivot scale $k_* = 0.05$ Mpc$^{-1}$,

$$\delta \hat{a} = \frac{H_*}{2\pi}$$

with the axion field $\hat{a} = \theta \chi_*$ and $\theta$ the initial misalignment angle.

• So, the perturbation in the misalignment angle is

$$\delta \theta = \frac{H_*}{2\pi \chi_*}.$$

• At the QCD transition, $a$ acquires a mass $m_a$ and starts oscillating coherently with initial amplitude $a = \theta f_a$ and energy density

$$\rho_a = \frac{1}{2} m_a^2 a^2.$$

• $\delta \theta$ then translates into a perturbation in the initial amplitude

$$\delta a = \delta \theta f_a = \frac{H_* f_a}{2\pi \chi_*},$$

yielding a perturbation $\delta n_a$ in the axion number density $n_a$.

• The resulting axion isocurvature perturbation is

$$S_a = \frac{\delta n_a}{n_a} = \frac{\delta \rho_a}{\rho_a} = 2 \frac{\delta a}{a} = \frac{H_*}{\pi \theta \chi_*},$$

and is completely uncorrelated with the curvature perturbation.
• The overall isocurvature perturbation generated by the axions is
\[ S = \frac{H^*_\pi}{\theta \chi^*_\pi} R_a, \]
where \( R_a \) is the axion fraction of cold dark matter.

• The Planck data indicate that the isocurvature fraction
\[ \beta_{\text{iso}}(k_*) = \frac{\mathcal{P}_{\text{II}}(k_*)}{\mathcal{P}_{\text{RR}}(k_*) + \mathcal{P}_{\text{II}}(k_*)} \lesssim 0.038 \text{ at } 95\% \text{ c.l.} \]
with \( \mathcal{P}_{\text{RR}} \) and \( \mathcal{P}_{\text{II}} \) the adiabatic and isocurvature power spectra.

• So, at 95\% c.l.,
\[ S \lesssim 0.1987 A_s(k_*)^{\frac{1}{2}} \simeq 9.35 \times 10^{-6}. \]

• \( R_a = \Omega_a h^2 / \Omega_{\text{CDM}} h^2 \) with \( \Omega_{\text{CDM}} h^2 \simeq 0.12 \) and
\[ \Omega_a h^2 \simeq 0.236 \left( \frac{f_a / \mathcal{N}}{10^{12} \text{ GeV}} \right)^{\frac{7}{6}} \langle \mathcal{N}^2 \theta^2 f(\mathcal{N} \theta) \rangle. \]

• \( \theta \in [-\pi/\mathcal{N}, +\pi/\mathcal{N}] \), where \( \mathcal{N} \) is the absolute value of the sum of the PQ charges of all fermionic color (anti)triplets.

• In the simplest models such as ours, \( \mathcal{N} = 6. \)

• \( f(\mathcal{N} \theta) \) accounts for the anharmonicity of the axion potential and the average \( \langle \mathcal{N}^2 \theta^2 f(\mathcal{N} \theta) \rangle \) is evaluated in the above interval.

• \( \mathcal{N} \) determines the \( Z_{\mathcal{N}} \) subgroup of \( U(1)_{\text{PQ}} \) which remains explicitly unbroken by instantons, but is spontaneously broken by \( \langle \mathcal{N} \rangle. \)

• This could produce disastrous walls, but \( U(1)_{\text{PQ}} \) is spontaneously broken during inflation and no walls problem arises.
• The average $\langle \mathcal{N}^2 \theta^2 f(\mathcal{N}\theta) \rangle \simeq 8.77$ and, for $f_a = 10^{11}$ GeV and $\theta = 0.3$, $R_a \simeq 0.14$.

• The Planck bound then implies

$$\chi^* \gtrsim 4.28 \times 10^{-4}.$$

• For $d_1 = 1$, this is satisfied if $\lambda \lesssim 0.3 \Rightarrow m_N \lesssim 1069$ GeV.

• With smaller $\lambda$’s we can get bigger $f_a$’s and, thus, bigger $R_a$’s.

• For $f_a = 4 \times 10^{11}$ GeV, $R_a \simeq 0.73$ and the bound is satisfied for $\chi^* \gtrsim 2.26 \times 10^{-3}$, which requires $\lambda \lesssim 0.01$ and $m_N \lesssim 570$ GeV.

• So the axion fraction of dark matter can be naturally sizable and axions may be detectable in future microwave cavity experiments.

5 Summary

• We provided a simple realistic SUSY hybrid inflation model intimately linking axion physics with the resolution of the $\mu$ problem.

• We showed that the PQ transition proceeds without problems.

• In particular, no unacceptably large axion isocurvature perturbations are generated.

• $n_s$ can satisfy the observational bounds, while $r$ is negligible.

• The axion domain walls are inflated away.

• The saxion, the complex $\bar{N}$ Higgs, and the axino remain always subdominant and decay well before the freeze out of the LSPs.

• So, the neutralino relic density is not affected and the axions and/or the lightest SUSY particle compose the dark matter.