# PECCEI-QUINN TRANSITION AND SUPERSYMMETRIC HYBRID INFLATION

## 1 Introduction

- SUSY hybrid inflation is a very promising inflation scenario.
- It predicts $n_{\rm s}$ in the observed range of 0.96-0.97.
- SUGRA corrections are under control for inflaton values $\ll m_{\rm P}$.
- Here, we merge SUSY hybrid inflation with axion physics.
- The intermediate PQ scale $f_{\rm a}$ can generate the MSSM $\mu$ term.
- Namely, the 'bare' $\mu$ term is forbidden through symmetries, but it arises from a higher dimensional superpotential term.
- The resulting $\mu \sim f_{\rm a}^2/m_{\rm P} \sim {\rm TeV}$, the desired magnitude.
- The tensor-to-scalar ratio $r$ is negligible in this simple model.
- Both the axion and the LSP would contribute to dark matter.
- We investigate how the PQ transition proceeds in a SUSY model with hybrid inflation in accord with observations.
- A potential problem is the generation of unacceptably large axion isocurvature perturbations.
- To avoid them the PQ field during inflation must be fairly large.
- This is achieved by higher order terms in the Kähler potential.
- They produce a negative ${\rm mass}^2$ for the PQ field thereby shifting its vacuum value to become large during inflation.
- Thus, the PQ field is large during inflation and inflaton oscillations.

- It then gradually drifts to the desired low energy PQ vacuum.
- So, the PQ symmetry is spontaneously broken already during inflation and remains so thereafter.
- The axion domain wall problem, which appears if the spontaneous breaking of $U(1)_{\rm PQ}$ occurs after inflation, does not arise.
- Our numerical study shows how the PQ field develops during inflation and inflaton oscillations until low energies.
- The system contains not only the saxion and the axion but also an extra complex Higgs field and a four component Dirac axino.
- We estimate the amplitude of the PQ oscillations when soft SUSY breaking takes over and calculate the ensuing abundances.
- The decay time and thermal density of the PQ states is estimated.

## 2 Model and the PQ transition

- Consider a SUSY model based on the left-right symmetric gauge group $G_{\rm LR} = SU(3)_{\rm c} \times SU(2)_{\rm L} \times SU(2)_{\rm R} \times U(1)_{B-L}$.
- The superfields are the gauge singlets $S$, $N$, $\bar{N}$, the Higgses $H = (1,1,2)_1$, $\bar{H} = (1,1,2)_{-1}$ causing $G_{\rm LR} \to G_{\rm SM}$, the electroweak Higgs $h = (1,2,2)_0$, and the matter superfields $q_i$, $q_i^c$, $l_i$, $l_i^c$.
- The model has a R symmetry $U(1)_{\rm R}$ with the superfields charges

$$R: S(2), N(0), \bar{N}(2), H, \bar{H}(0), h(1), q_i, l_i(0), q_i^c, l_i^c(1).$$

- Also, a PQ symmetry $U(1)_{\rm PQ}$ with the superfields charges

$$PQ: S(0), N(-1), \bar{N}(3), H, \bar{H}(0), h(1), q_i, l_i(-1), q_i^c, l_i^c(0).$$

- The symmetries allow $\bar{H}\bar{H}l_i^c l_j^c$ for right-handed neutrino masses.
- The 'bare' $\mu$ term $h^2$ is forbidden, but the term $N^2h^2$ is allowed and generates a $\mu$ term of the desired magnitude.
- The soft SUSY breaking terms break $U(1)_{\rm R}$ explicitly to its $Z_2$ subgroup under which $h$, $q_i^c$, and $l_i^c$ are odd.
- This $Z_2$, combined with the $Z_2$ center of $SU(2)_{\rm L}$ provides the well-known $Z_2^{\rm mp}$ under which the matter fields are odd.
- So, the apparent spontaneous breaking of the $Z_2$ subgroup of $U(1)_{\rm R}$ does not lead to domain walls.
- This $Z_2$ is replaced by the equivalent $Z_2^{\rm mp}$, which is unbroken.
- $U(1)_{\rm PQ}$ is broken explicitly by instantons to $Z_6$, which breaks spontaneously during inflation and the walls are inflated away.
- The symmetries imply perturbative baryon number conservation.
- The superpotential relevant for inflation and the PQ system is

$$W = \kappa S \left(M^2 - H\bar{H}\right) + \frac{\lambda}{m_{\rm P}} N^3 \bar{N}.$$

- The Kähler potential is taken to be

$$\begin{aligned} K &= |S|^2 + |H|^2 + |\bar{H}|^2 + |N|^2 + |\bar{N}|^2 + \frac{\alpha}{4m_{\rm P}^2}|S|^4 + \\ & \left(c_1|S|^2 + c_2|H|^2 + c_3|\bar{H}|^2 + c_4\left(H\bar{H} + h.c.\right)\right)\frac{|N|^2}{m_{\rm P}^2} + \\ & \left(c_5|S|^2 + c_6|H|^2 + c_7|\bar{H}|^2 + c_8\left(H\bar{H} + h.c.\right)\right)\frac{|\bar{N}|^2}{m_{\rm P}^2}. \end{aligned}$$

- $M$ is a superheavy mass and $\kappa$, $\lambda$, $\alpha$, $c_i$ are dimensionless constants taken real with $\alpha > 0$, $c_1 > 1$, $c_2$, $c_3 \geq 1$, $c_5$, $c_6$, $c_7 < 1$.

- Expand the F-term potential in powers of $m_{\rm P}^{-1}$ up to 2nd order:

$$
\begin{aligned}
V \;=\; & \kappa^2|M^2 - H\bar{H}|^2\left(1+|H|^2+|\bar{H}|^2-\alpha|S|^2 - \right.\\
& \left.(c_1-1)|N|^2+(1-c_5)|\bar{N}|^2\right) + \\
& \kappa^2|S|^2\left(|H|^2+|\bar{H}|^2\right)\left(1+|S|^2+|H|^2+|\bar{H}|^2\right) + \\
& 4\kappa^2|S|^2|H|^2|\bar{H}|^2 - 2\kappa^2M^2|S|^2\left(H\bar{H}+h.c.\right) - \\
& \kappa^2|S|^2\left((c_3-1)|H|^2+(c_2-1)|\bar{H}|^2\right)|N|^2 + \\
& \kappa^2|S|^2\left((1-c_7)|H|^2+(1-c_6)|\bar{H}|^2\right)|\bar{N}|^2 + \\
& \lambda^2|N|^4\left(|N|^2+9|\bar{N}|^2\right) + \\
& m_{3/2}^2\left(\varepsilon_1|N|^2+\varepsilon_2|\bar{N}|^2\right)+m_{3/2}\left(\lambda A N^3\bar{N}+h.c.\right).
\end{aligned}
$$

- We included here only the soft terms for $N$, $\bar{N}$ and put $m_{\rm P}=1$. We set $A=0$ and take $\varepsilon_2>0$.
- For $c_5$, $c_6$, $c_7<1$, the minimum of $V$ is always at $\bar{N}=0$ and, thus, we safely set $\bar{N}=0$ throughout the discussion.
- To achieve spontaneous breaking of the PQ symmetry, we choose $\varepsilon_1<0$ and absorb it redefining $m_{3/2}^2$.
- Take the D-flat direction $H=\bar{H}^*$ containing the SUSY vacuum.
- By $U(1)_{\rm R}$, $U(1)_{\rm PQ}$, $U(1)_{B-L}$ rotations, we make $S$, $N$, $H$, $\bar{H}$ real and define almost canonically normalized real scalars $\sigma$, $\chi$, $h$:

$$S=\frac{\sigma}{\sqrt{2}},\quad N=\frac{\chi}{\sqrt{2}},\quad H=\bar{H}=\frac{h}{2}.$$

- In terms of $\sigma$, $\chi$, $h$, the potential is

$$
\begin{aligned}
V \;=\; & \kappa^2\left(M^2-\frac{h^2}{4}\right)^2\left(1+\frac{h^2}{2}-\alpha\frac{\sigma^2}{2}-d_1\frac{\chi^2}{2}\right)+\frac{\lambda^2}{8}\chi^6 \\
& \frac{\kappa^2}{4}\sigma^2h^2\left(1+\frac{\sigma^2}{2}+h^2-2M^2-d_2\frac{\chi^2}{2}\right)-\frac{1}{2}m_{3/2}^2\chi^2,
\end{aligned}
$$

where $d_1 \equiv c_1 - 1 > 0$, $d_2 \equiv (c_2 + c_3)/2 - 1 \geq 0$.

- The $\chi$-dependent part of $V$ may be written as

$$V_\chi = -\frac{1}{2} m_\chi^2 \chi^2 + \frac{\lambda^2}{8} \chi^6,$$

where

$$m_\chi^2 = d_1\, \kappa^2 \left( M^2 - \frac{h^2}{4} \right)^2 + d_2\, \frac{\kappa^2}{4} \sigma^2 h^2 + m_{3/2}^2.$$

- $V_\chi$ is minimized at $\chi = \chi_{\rm m}$:

$$|\chi_{\rm m}| = \sqrt{2} \left( \frac{m_\chi^2}{3\lambda^2} \right)^{\frac{1}{4}}.$$

- $c_1 > 1$, $c_2$, $c_3 \geq 1 \Rightarrow m_\chi^2 > 0$ and the minimum during inflation and inflaton oscillations is shifted away from $\chi = 0$.
- This is crucial for controlling the axion isocurvature perturbations and avoiding the axion domain wall problem.
- For $\sigma > \sigma_{\rm c} \simeq \sqrt{2} M$, the hybrid inflation path at $h = 0$ is stable.
- The inflationary potential is

$$V \overset{h=0}{=} \kappa^2 M^4 \left( 1 - \alpha \frac{\sigma^2}{2} \right) + V_\chi \simeq \kappa^2 M^4, \quad \text{and} \quad m_\chi^2 \simeq d_1 \kappa^2 M^4.$$

- So, $|\chi_{\rm m}|$ during inflation becomes

$$|\chi_{\rm inf}| \simeq \sqrt{2} \left( \frac{d_1 \kappa^2 M^4}{3\lambda^2} \right)^{\frac{1}{4}}.$$

- Since $m_\chi \sim H_{\rm inf}$, $\chi$ is expected to quickly reach the above value.
- To the inflationary potential we add the radiative corrections

$$V_{\rm rad} = \kappa^2 M^4 \left(\frac{\delta_h}{2}\right) \ln \frac{\sigma^2}{\sigma_{\rm c}^2}, \quad \delta_h = N_h \frac{\kappa^2}{8\pi^2}$$

where $N_h = 2$ is the dimensionality of $H$, $\bar{H}$.

- The potential during inflation is then

$$V_{\rm inf} \simeq \kappa^2 M^4 \left(1 + \frac{\delta_h}{2} \ln \frac{\sigma^2}{\sigma_{\rm c}^2} - \alpha \frac{\sigma^2}{2}\right) \simeq \kappa^2 M^4.$$

- The slow-roll parameters are

$$\epsilon \equiv \frac{1}{2}\left(\frac{V_{\rm inf}'}{V_{\rm inf}}\right)^2 \simeq \frac{1}{2}\alpha\delta_h \frac{(1-x)^2}{x}, \quad \eta \equiv \frac{V_{\rm inf}''}{V_{\rm inf}} \simeq -\alpha\frac{1+x}{x},$$

$$\xi \equiv \left(\frac{V_{\rm inf}'}{V_{\rm inf}}\right)\left(\frac{V_{\rm inf}'''}{V_{\rm inf}}\right) \simeq 2\alpha^2 \frac{1-x}{x^2}, \quad \text{with} \quad x \equiv \frac{\alpha\sigma^2}{\delta_h} < 1.$$

- Inflation ends when $\sigma$ reaches the value $\sigma_{\rm end}$ with

$$\sigma_{\rm end}^2 \simeq \max\{2M^2, \delta_h\},$$

depending on whether inflation ends with a waterfall or smoothly.

- The number $\mathbf{N}(x_{\rm in})$ of e-foldings between $\sigma = \sigma_{\rm in}$ and $\sigma = \sigma_{\rm end}$ corresponding to $x_{\rm in}$ and $x_{\rm end}$ is

$$\mathbf{N}(x_{\rm in}) \simeq \frac{1}{2\alpha} \ln \frac{1 - x_{\rm end}}{1 - x_{\rm in}}.$$

- So, if $x = x_*$ at horizon exit of the pivot scale $k_* = 0.05~{\rm Mpc}^{-1}$,

$$\mathbf{N}_* \equiv \mathbf{N}(x_*) \simeq \frac{1}{2\alpha} \ln \frac{1}{1 - x_*}$$

- The scalar spectral index $n_{\rm s}$ is

$$n_{\rm s} \simeq 1 + 2\eta_* \simeq 1 - 2\alpha \frac{1+x_*}{x_*} \simeq 1 - \frac{1}{\mathbf{N}_*}\left(\frac{1+x_*}{x_*}\ln\frac{1}{1-x_*}\right).$$

- Its running $\alpha_{\rm s} \equiv dn_{\rm s}/d\ln k$ and is the inflationary $V$ are

$$\alpha_{\rm s} \simeq -2\xi_* \simeq -4\alpha^2 \frac{1-x_*}{x_*^2}, \quad V = 24\pi^2\epsilon_* A_{\rm s},$$

  $A_{\rm s}$ is the scalar power spectrum amplitude at horizon exit of $k_*$.

- Many SUSY inflation scenarios predict $n_{\rm s} \simeq 1 - 1/\mathbf{N}_* \simeq 0.98$.
- A modified relation $n_{\rm s} \simeq 1-2/\mathbf{N}_*$, though, is in better agreement with observations and is easily obtain in our model for $x_* \simeq 0.5$.
- Indeed, $x_* = 0.5$ and $\mathbf{N}_* = 52 \Rightarrow \alpha \simeq 1/150$, $n_{\rm s} \simeq 0.96$, $\alpha_{\rm s} \simeq -3.56 \times 10^{-4}$, $M \simeq 2.17 \times 10^{-3}$ for $A_{\rm s} = 2.215 \times 10^{-9}$.
- We now put $\kappa = 0.01 \Rightarrow |\sigma_*| \simeq 0.01378$, $x_{\rm end} \simeq 0.0248$, $\epsilon_* \simeq 4.22\times10^{-9}$, and the tensor-to-scalar ratio $r = 16\epsilon_* \simeq 6.76\times10^{-8}$.
- So gravity waves are not observable.
- $V$ during inflaton oscillations $\simeq E/2$ with $E$ being the energy density of the rapidly oscillating massive fields $\sigma$ and $h$.
- So the position of the minimum of $V$ is

$$|\chi_{\rm m}| \simeq \sqrt{2}\left(\frac{d\,E/2 + m_{3/2}^2}{3\lambda^2}\right)^{\frac{1}{4}} \quad \text{with} \quad d = d_1 = d_2$$

- As $d\,E/2$ approaches $m_{3/2}^2$, $|\chi_{\rm m}|$ reaches smoothly the value

$$|\langle\chi\rangle| = \sqrt{2}|\langle N\rangle| \equiv f_{\rm a} = \sqrt{2}\left(\frac{m_{3/2}^2}{3\lambda^2}\right)^{\frac{1}{4}},$$

  which determines the PQ breaking scale $f_{\rm a}$ (axion decay constant).

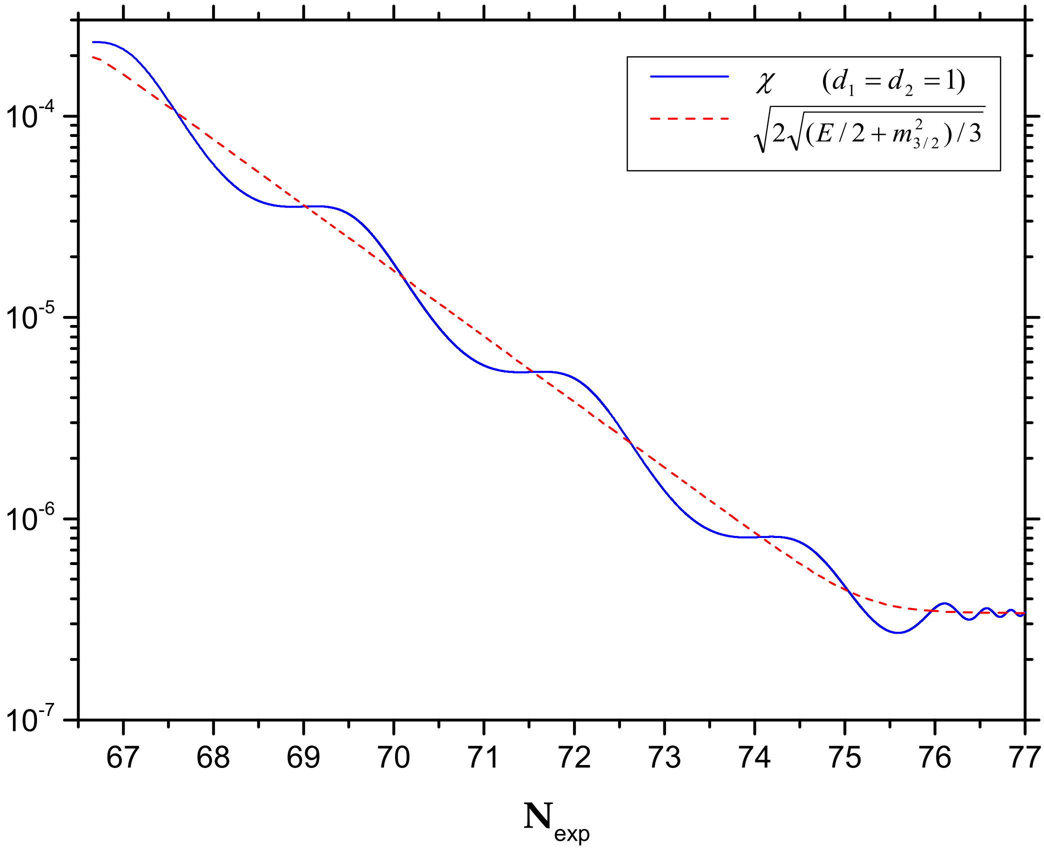


Figure 1: Postinflationary evolution of $\chi$ versus the number of e-foldings $\mathbf{N}_{\rm exp}$ for $m_{3/2} = 10^{-13}$, $d_1 = d_2 = 1$. The moving position of the minimum of the potential is also depicted.

- To confirm our understanding for the postinflationary evolution of $\chi$, we solve numerically the DEs for the dynamics of the system.
- We take $\kappa = 0.01$, $\lambda = 1$, $\alpha = 1/150$, $M = 2.17 \times 10^{-3}$ and the initial conditions $\sigma = 0.015$, $\dot{\sigma} = -1.86 \times 10^{-12}$, $h = 10^{-8}$, $\dot{h} = 0$, $\chi = 10^{-5}$, $\dot{\chi} = 0$.
- The initial $\dot{\sigma}$ is the velocity of $\sigma$ during inflation found numerically.
- We followed the evolution for 77 e-foldings out of which about 66.7 correspond to inflationary expansion.
- As an example, we show $\chi$ and $|\chi_{\rm m}|$ as functions of $\mathbf{N}_{\rm exp}$ only for the period after inflation for $m_{3/2} = 10^{-13}$, $d_1 = d_2 = 1$.
- $\mathbf{N}_{\rm exp}$ is from $t = 0$ where the initial conditions were imposed.

- We see that $\chi$ oscillates around the moving minimum $\chi_{\rm m}$.
- After 76 e-foldings, $\chi$ reaches the low-energy PQ minimum and the amplitude of oscillations is severely reduced.
- We studied the evolution of $\chi$ for various choices of $m_{3/2}$, $d_1$, $d_2$ including $d_2 \leq 0$. The emerging picture is basically the same.
- So our numerical findings support our theoretical expectations.

## 3 Relic density of the PQ fields

- From $\delta W_1 = \lambda N^3 \bar{N}$, we obtain the potential in global SUSY:
$$V = \lambda^2 |N|^4 (|N|^2 + 9|\bar{N}|^2) - m_N^2 |N|^2 + m_{\bar{N}}^2 |\bar{N}|^2.$$
Here $-m_N^2 = \varepsilon_1 m_{3/2}^2$, $m_{\bar{N}}^2 = \varepsilon_2 m_{3/2}^2$ are the soft masses squared of $N$, $\bar{N}$ and the soft A term is set to zero.
- $V$ has a local maximum at $|N| = |\bar{N}| = 0$ and the absolute minimum at
$$|N| = \left(\frac{m_N^2}{3\lambda^2}\right)^{\frac{1}{4}} \equiv \langle N \rangle, \qquad |\bar{N}| = 0,$$
corresponding to the low-energy PQ vacuum.
- Writing $N = \langle N \rangle + \delta N$ with $\delta N = (\delta\chi + ia)/\sqrt{2}$ in $V$, we obtain a saxion $\delta\chi$ with $m_{\delta\chi}^2 = 4m_N^2$, a massless axion $a$, and an extra complex scalar $\bar{N} = (\bar{\chi} + i\bar{a})\sqrt{2}$ with $m_{\bar{\chi}}^2 = 3m_N^2 + m_{\bar{N}}^2$.
- Saxion decays to MSSM Higgsinos $\tilde{h}_u$, $\tilde{h}_d$ via the Yukawa coupling
$$-2\sqrt{2}\,\frac{\mu}{f_{\rm a}}\,\delta N \tilde{h}_u \tilde{d}_d + h.c.,$$
from $\delta W_2 = \frac{1}{2}\lambda_\mu N^2 h^2$, which also gives the $\mu$ term with $\mu = \lambda_\mu \langle N \rangle^2$ after the PQ breaking.

- The saxion decay width is estimated to be

$$\Gamma_{\rm d} \simeq \frac{1}{\pi}\left(\frac{\mu}{f_{\rm a}}\right)^2 m_{\delta\chi}.$$

- The energy density $\rho_{\delta\chi}$ of coherent saxion oscillations behaves like pressureless matter and, thus, when saxion decays at $t_{\rm d} = \Gamma_{\rm d}^{-1}$, is

$$\rho_{\delta\chi}^{\rm d} = \rho_{\delta\chi}^{\rm r}\left(\frac{t_{\rm r}}{t_{\rm d}}\right)^{\frac{3}{2}},$$

where $\rho_{\delta\chi}^{\rm r}$ is the energy density at the time $t_{\rm r}$ of reheating.

- The initial amplitude of saxion oscillations at $t_{\rm osc} \sim m_{\delta\chi}^{-1}$ is $\sim f_{\rm a}$.

- So, the initial energy density is

$$\rho_{\delta\chi}^{\rm osc} \sim \frac{1}{2} m_{\delta\chi}^2 f_{\rm a}^2 \quad \Rightarrow \quad \rho_{\delta\chi}^{\rm r} \sim \frac{1}{2} m_{\delta\chi}^2 f_{\rm a}^2 \left(\frac{t_{\rm osc}}{t_{\rm r}}\right)^2,$$

since the universe is matter dominated between $t_{\rm osc}$ and $t_{\rm r}$.

- We then find that

$$\rho_{\delta\chi}^{\rm d} \sim \frac{1}{2\pi^2} m_{\delta\chi}^2 f_{\rm a}^2 \left(\frac{t_{\rm d}}{t_{\rm r}}\right)^{\frac{1}{2}} \left(\frac{\mu}{f_{\rm a}}\right)^4.$$

- For $f_{\rm a} = 10^{11}$ GeV and $\mu = m_{\delta\chi} = 1$ TeV, we find $t_{\rm d} \simeq 3.14 \times 10^{13}$ GeV$^{-1}$ and, for $T_{\rm r} = 10^9$ GeV, we get $t_{\rm r} \simeq 0.24$ GeV$^{-1}$.

- We then obtain

$$\rho_{\delta\chi}^{\rm d} \simeq 58 \text{ GeV}^4 \ll \rho_{\rm rad}^{\rm d} \simeq 4.5 \times 10^9 \text{ GeV}^4,$$

and the energy density of saxion oscillations never dominates.

- We also find $T_{\rm d} \simeq 110$ GeV $>$ the neutralino freeze-out temperature $T_{\rm f} \simeq m_{\rm LSP}/25$ if the neutralino mass $m_{\rm LSP} \lesssim 1.65$ TeV.

- So, in this case, the neutralino relic abundance is not affected.
- Thermal saxion production is dominated by processes involving again the saxion-Higgsino-Higgsino coupling.
- The thermal saxion abundance $Y_{\delta\chi} \equiv \frac{n_{\delta\chi}}{s}$ is similar to that of $\tilde{a}$

$$Y_{\tilde{a}} \simeq 10^{-5} \left( \frac{f_{\rm a}}{10^{11}~\rm GeV} \right)^{-2} \left( \frac{\mu}{10^{3}~\rm GeV} \right)^{2} \simeq 10^{-5}.$$

- Thus, the thermal saxion energy density $\rho^{\rm d}_{\delta\chi}$ at saxion decay is

$$\frac{\rho^{\rm d}_{\delta\chi}}{\rho^{\rm d}_{\rm rad}} \simeq \frac{4}{3}\frac{Y_{\delta\chi} m_{\delta\chi}}{T_{\rm d}} \sim 10^{-4} \ll 1$$

  and the thermal saxions also remain always subdominant.
- The oscillations of complex Higgs $\bar{N}$ are, as shown numerically, of vanishing amplitude and do not contribute to the energy density.
- Thermal production of $\bar{N}$'s involves the coupling

$$2\sqrt{2}\,\frac{\mu}{f_{\rm a}}\frac{m_{\tilde{a}}}{m_{\bar{\chi}}} m_{\bar{\chi}} \bar{N} h_u h_d + h.c.,$$

  where $m_{\tilde{a}} = \sqrt{3} m_N$ is the axino mass.
- $\bar{N}$ decays into a pair of ordinary Higgses via the same coupling.
- So, $\bar{N}$ also decays well before the freeze out of neutralinos remaining subdominant.
- The superpotential term $\delta W_1$ gives rise to the Yukawa coupling

$$-3\lambda \langle N \rangle^2 \tilde{N} \tilde{\bar{N}} + h.c.$$

  yielding a four component axino $\tilde{a}$ with Dirac mass $m_{\tilde{a}} = \sqrt{3} m_N$.
- The axino decays into an ordinary Higgs and a Higgsino, again well before the neutralino freeze out and remains subdominant.

- So, saxion, $\bar{N}$, $\tilde{a}$ do not affect the universe in any essential way.
- Thermal axions contribution to the effective number of $\nu$ is tiny.

## 4 Axion Isocurvature Perturbations

- The axion during inflation is massless and thus acquires a perturbation which, at horizon exit of the pivot scale $k_* = 0.05\ \mathrm{Mpc}^{-1}$,

$$\delta\hat{a} = \frac{H_*}{2\pi}$$

  with the axion field $\hat{a} = \theta\chi_*$ and $\theta$ the initial misalignment angle.
- So, the perturbation in the misalignment angle is

$$\delta\theta = \frac{H_*}{2\pi\chi_*}.$$

- At the QCD transition, $a$ acquires a mass $m_a$ and starts oscillating coherently with initial amplitude $a = \theta f_\mathrm{a}$ and energy density

$$\rho_a = \frac{1}{2} m_a^2 a^2.$$

- $\delta\theta$ then translates into a perturbation in the initial amplitude

$$\delta a = \delta\theta f_\mathrm{a} = \frac{H_* f_\mathrm{a}}{2\pi\chi_*},$$

  yielding a perturbation $\delta n_a$ in the axion number density $n_a$.
- The resulting axion isocurvature perturbation is

$$\mathcal{S}_a = \frac{\delta n_a}{n_a} = \frac{\delta\rho_a}{\rho_a} = 2\frac{\delta a}{a} = \frac{H_*}{\pi\theta\chi_*},$$

  and is completely uncorrelated with the curvature perturbation.

- The overall isocurvature perturbation generated by the axions is

$$\mathcal{S} = \frac{H_*}{\pi\theta\chi_*} R_a,$$

where $R_a$ is the axion fraction of cold dark matter.

- The Planck data indicate that the isocurvature fraction

$$\beta_{\rm iso}(k_*) \equiv \frac{\mathcal{P}_{\mathcal{II}}(k_*)}{\mathcal{P}_{\mathcal{RR}}(k_*) + \mathcal{P}_{\mathcal{II}}(k_*)} \lesssim 0.038 \quad \text{at} \quad 95\% \; c.l.$$

with $\mathcal{P}_{\mathcal{RR}}$ and $\mathcal{P}_{\mathcal{II}}$ the adiabatic and isocurvature power spectra.

- So, at $95\%$ c.l.,

$$\mathcal{S} \lesssim 0.1987\, A_{\rm s}(k_*)^{\frac{1}{2}} \simeq 9.35 \times 10^{-6}.$$

- $R_a = \Omega_a h^2/\Omega_{\rm CDM} h^2$ with $\Omega_{\rm CDM} h^2 \simeq 0.12$ and

$$\Omega_a h^2 \simeq 0.236 \left(\frac{f_{\rm a}/\mathcal{N}}{10^{12}\ {\rm GeV}}\right)^{\frac{7}{6}} \langle \mathcal{N}^2\theta^2 f(\mathcal{N}\theta)\rangle.$$

- $\theta \in [-\pi/\mathcal{N}, +\pi/\mathcal{N}]$, where $\mathcal{N}$ is the absolute value of the sum of the PQ charges of all fermionic color (anti)triplets.
- In the simplest models such as ours, $\mathcal{N} = 6$.
- $f(\mathcal{N}\theta)$ accounts for the anharmonicity of the axion potential and the average $\langle \mathcal{N}^2\theta^2 f(\mathcal{N}\theta)\rangle$ is evaluated in the above interval.
- $\mathcal{N}$ determines the $Z_{\mathcal{N}}$ subgroup of $U(1)_{\rm PQ}$ which remains explicitly unbroken by instantons, but is spontaneously broken by $\langle N\rangle$.
- This could produce disastrous walls, but $U(1)_{\rm PQ}$ is spontaneously broken during inflation and no walls problem arises.

- The average $\langle \mathcal{N}^2\theta^2 f(\mathcal{N}\theta)\rangle \simeq 8.77$ and, for $f_{\rm a} = 10^{11}$ GeV and $\theta = 0.3$, $\mathsf{R}_a \simeq 0.14$.
- The Planck bound then implies

$$\chi_* \gtrsim 4.28 \times 10^{-4}.$$

- For $d_1 = 1$, this is satisfied if $\lambda \lesssim 0.3 \Rightarrow m_N \lesssim 1069$ GeV.
- With smaller $\lambda$'s we can get bigger $f_{\rm a}$'s and, thus, bigger $R_a$'s.
- For $f_{\rm a} = 4 \times 10^{11}$ GeV, $R_a \simeq 0.73$ and the bound is satisfied for $\chi_* \gtrsim 2.26 \times 10^{-3}$, which requires $\lambda \lesssim 0.01$ and $m_N \lesssim 570$ GeV.
- So the axion fraction of dark matter can be naturally sizable and axions may be detectable in future microwave cavity experiments.

## 5 Summary

- We provided a simple realistic SUSY hybrid inflation model intimately linking axion physics with the resolution of the $\mu$ problem.
- We showed that the PQ transition proceeds without problems.
- In particular, no unacceptably large axion isocurvature perturbations are generated.
- $n_{\rm s}$ can satisfy the observational bounds, while $r$ is negligible.
- The axion domain walls are inflated away.
- The saxion, the complex $\bar{N}$ Higgs, and the axino remain always subdominant and decay well before the freeze out of the LSPs.
- So, the neutralino relic density is not affected and the axions and/or the lightest SUSY particle compose the dark matter.