Non-susy model relates CDM directly to the observed primordial magnetic fields

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0. Abstract

A primordial geon $\mathcal{G}$, namely an exact particle-like solution in Einstein’s gravity coupled to sourceless Maxwell content $F$, is presented and examined as a non-susy candidate element for CDM. $\mathcal{G}$ is a non-singular pp-wave manifold with $S^3 \times \mathbb{R}$ topology and a length scale of $L^2 = (\kappa Q)^2$, which emerges as an overall conformal factor in $\mathcal{G}$’s line element. $Q$ is the effective charge of a primordial $Q/r^2$ magnetic monopole as the basic field in $F$. These aspects, together with the potentially large effective mass ($\sim L^{-1}$) of $\mathcal{G}$, seem to qualify it as an element for CDM. Thus, the latter may also relate directly to the observed primordial magnetic fields.
1. Reminders on “primordial”

1. A “[classical] primordial field” is any field, sourceless by its own FEs, coupled to Einstein’s gravity in a smooth manifold $M$ with $\partial M = \emptyset$.

2. Primordial as in “primordial gravitational waves”, refers to any entity which participates in the early environment of quantum gravity.
2. Conditions on $\mathcal{G}$ as a candidate element for CDM

1. $\mathcal{G}$ must have ALF-supporting topology
   (cf. also next item and compare with Taub-NUT).
2. $\mathcal{G}$ cannot have any mathematical singularity
   (e.g., R-N types would collapse to black holes).
3. $\mathcal{G}$’s length scale $L$ should be naturally small
   (e.g., as a specific $L$ imposed by the FEs).
4. Maxwell’s field-content must be primordial
   ($dF = d \ast F = 0$ must hold throughout $\mathcal{G}$).
5. $\mathcal{G}$ should be a pp-wave manifold, so with
   $k^2 = 0, \nabla k = 0$, Gibbons’ observation applies.
6. $\mathcal{G}$ should be a solitonic particle-like configuration.
3. Topology and metric of $\mathcal{G}$

$\mathcal{G}$’s topology is $S^3 \times \mathbb{R}$ (comment vs Taub-NUT); the metric and symmetries can be expressed as

$$ds^2 = -L^2 \left( \sigma^3 + 2du \right) \sigma^3 + r^2 d\Omega^2 \quad (0.1)$$

$$d\Omega^2: = (\sigma^1)^2 + (\sigma^2)^2 = du^2 + \sin^2 \vartheta \ d\varphi^2$$

namely a Bianchi-type IX with left-$SU(2)$ invariant basis of 1-forms $\sigma^i$ and dual $\Sigma_i$ ($< \sigma^i | \Sigma_j > \delta^i_j$) in

$$d\sigma^i = -\frac{1}{2} \epsilon^i_{jk} \sigma^j \wedge \sigma^k \quad \leftrightarrow \quad [\Sigma_j, \Sigma_k] = \epsilon^i_{jk} \Sigma_i.$$

The unknown $r = r(u)$ will be given by the field equations as $r^2 = r_o^2 + L^2 u^2$, together with $r_o \sim L$, $L = 8\pi G_N Q$, $m_G \sim 1/L$, $Q =$ magnetic charge.
4. Cartan comoving frames $\Theta_\alpha = \Theta^\mu_\alpha \Sigma_\mu$, $\theta^\alpha = \theta_\mu^\alpha \sigma^\mu$,
give $ds^2 = \eta_{\alpha\beta} \theta^\alpha \theta^\beta$ everywhere in $G$. The basis of
covectors (1-forms) $< \theta^\alpha |$ is dual to the basis of row
vectors $| \Theta_\alpha >$ (with vierbeins $\theta_\mu^\alpha(u)$ and inverse
$\Theta^\alpha_\mu(u)$ for $< \theta^\alpha | \Theta_\beta > = \delta^\alpha_\beta$). We can thus find, e.g.,
$du = - L^{-1} \sinh \Phi \theta^0 + L^{-1} \cosh \Phi \theta^3$ from $\theta_\mu^\alpha(u)$, or
$\nabla \partial u = 0$ for $\partial u = Le^\Phi (\Theta^0_0 + \Theta^3_3)$ from $\Theta^\mu_\alpha(u)$, with
\[
\begin{bmatrix}
\theta^0 \\
\theta^1 \\
\theta^2 \\
\theta^3
\end{bmatrix} =
\begin{bmatrix}
Le^\Phi & 0 & 0 & L \cosh \Phi \\
0 & r & 0 & 0 \\
0 & 0 & r & 0 \\
Le^\Phi & 0 & 0 & \sinh \Phi
\end{bmatrix}
\begin{bmatrix}
\sigma^0 \\
\sigma^1 \\
\sigma^2 \\
\sigma^3
\end{bmatrix},
\]
etc. $\Phi =$ const phase parameter of a global isometry.
5. The content of the Maxwell sector of $\mathcal{G}$

From the E-H action (with $\kappa^2 = 8\pi G_N$)

$$\mathcal{L} = -\frac{1}{\kappa^2} \varepsilon^\beta_\alpha \wedge \mathcal{R}_\beta^\alpha + F \wedge *F$$

$F$ follows from integration of $dF = d*F = 0$, valid everywhere in $\mathcal{G}$. Geometry and symmetries of $\mathcal{G}$ restrict the $E, B$ fields of $F$ along the “3 axis” as $F = -E\theta^0 \wedge \theta^3 + B\theta^1 \wedge \theta^2$, $*F = B\theta^0 \wedge \theta^3 + E\theta^1 \wedge \theta^2$.

With a prime for $d/du$, $dF = 0$, $d*F = 0$ give

$$(r^2B)' + L^2E = 0 \text{ plus } (r^2E)' - L^2B = 0,$$

hence the first integral:

$$E^2 + B^2 = Q^2/r^4$$

($Q \neq 0$ stands for a constant magnetic charge).
5. The full electromagnetic content of $\mathcal{G}$

Our earlier result on $F = - E \theta^0 \wedge \theta^3 + B \theta^1 \wedge \theta^2$:

$$(r^2 B)' + L^2 E = 0, \quad (r^2 E)' - L^2 B = 0,$$

which gave us as a first integral the result:

$$E^2 + B^2 = Q^2 / r^4,$$

now provides the full EM content in $\mathcal{G}$ as:

$$F = - \frac{2Qr_0 \sqrt{r^2 - r_o^2}}{r^4} \theta^0 \wedge \theta^3 + \left( \frac{Q}{r^2} - \frac{2Qr_o^2}{r^4} \right) \theta^1 \wedge \theta^2.$$

The dominant $Q/r^2$ magnetic monopole and dipole electric or quadrapole magnetic fields are primordial, so the respective sources are all effective. As with any electrovacuum, the above result accepts charge rotation, even though $Q$ is only an effective charge.
6. The $G = S_− \lor S_+$ structure

The $G$ manifold clearly has no boundary ($\partial G = \emptyset$), while its $G = S_− \lor S_+$ structure is essentially determined by $r^2 = r_o^2 + L^2u^2$ (as in Taub-NUT), illustrated below with two dimensions suppressed:
7. The $\mathcal{G}/2 = \mathcal{S}_+$ manifold and its boundary

The $\mathcal{G}/2 = \mathcal{S}_+$ manifold must carry real mass, EM charge, etc, on its $r = r_o \ (u = 0)$ locus of $S^2[r_o]$ on $\partial \mathcal{S}_+$. This is a physical singularity (geodesics end on $\partial \mathcal{S}_+$), as illustrated below:

![Diagram showing the $\mathcal{S}_+ = \mathcal{G}/2$ in the context of the $S^2[r]$ boundary](image)
8. Spin, effective mass and angular momentum of $G$

Integration of the EM energy density gives us:

$$m(r) = \int_{u=0}^{u(r)} \rho^{(em)} \theta^1 \wedge \theta^2 \wedge \theta^3$$

$$= \sqrt{2\pi}Q^2 \int_{r_o}^{r} d\sqrt{r^2 - r_o^2}/r^2$$

so the effective mass $m_G$ of $G$ is:

$$m_G = 2\sqrt{2\pi^2}Q^2/r_o.$$ 

Had the geometry allowed the $r_o = 0$ value, the $r \to \infty$ limit would reproduce the notorious (here a would-be-disastrous) result of a diverging $m_G$.

It is now straightforward to also calculate vorticity and therefore angular momentum (spin) of $G$. 
9. What we have found...

Each one of the $\mathcal{G}$ and $\mathcal{G}/2 = S_+$ manifolds has only one arbitrary constant as a free parameter, namely an electric or magnetic charge $Q$ in units which allow charge rotation. As geonic models, $\mathcal{G}$ and $S_+$ involve aspects which could survive quantization, with $\mathcal{G}$ being more appropriate to model the magnetic field of a primordial non-singular monopole; $S_+$ on the other hand solves the long-standing problem of a Coulomb $Q/r^2$ field without the $r = 0$ singularity; $S_+$ carries actual charge $Q$ (plus mass, spin, etc) on the $r = r_o$ locus (which is actually an $S^2$ on the $\partial S_+$ boundary).
10. ...and what could be a next step.

In the context of the present development, the standard model of cosmology should be upgraded, not with extra parameters, but rather with input from quantum gravity and primordial agents, plus an initially strong primordial magnetic field.

A toy model would involve a fluid of $G$-particles coupled to a large primordial magnetic field $B$. The model could somehow be incited or triggered to an inflationary start, and then left alone to end-up as a dust universe with a predictably small $B$ field.