Gravitational Waves & Leptogenesis From Higgs Inflation in Supergravity

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Based on:


Outline

Variants of Non-Minimal Higgs Inflation (non-MHI)

From Pure to Kinetically Modified non-Minimal Higgs Inflation
Inflation Analysis

The SUGRA Embedding

The (Semi)Logarithmic Kähler Potential
Softly Broken Shift Symmetry For Higgs Fields

Building A B – L GUT

Beyond MSSM With Several Consequences
The Inflationary Scenario

Post-Inflationary Evolution

Inflaton Decay & non-Thermal Leptogenesis
Conclusions

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The General Framework

- Our starting point is the action in the Jordan Frame (JF) of a Higgs field $\phi$ non-minimally coupled to the Ricci Scalar curvature, $\mathcal{R}$, through a frame function $f_R(\phi)$. This is:

$$S = \int d^4 x \sqrt{-g} \left( -\frac{1}{2} f_R(\phi) \mathcal{R} + \frac{f_K(\phi)}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V_{HI}(\phi) \right), \quad \text{where} \quad V_{HI} = \lambda^2 (\phi^2 - M^2)^2 / 16$$

is the potential of $\phi$. Also $g$ is the determinant of the background metric and we allow for a kinetic mixing through the function $f_K(\phi)$.

- At the vacuum, $\phi = M \ll 1$ so that $f_R(\langle \phi \rangle) \simeq 1$ (in reduced Planck units with $m_P = 1$) to guarantee the ordinary Einstein Gravity at low energy.

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**The General Framework**

- **Our Starting Point is The Action in the Jordan Frame (JF) Of A Higgs Field φ non-Minimally Coupled to the Ricci Scalar Curvature, R, Through A Frame Function f_R(φ). This is:**
  \[
  S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} f_R(\phi)R + \frac{f_K(\phi)}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V_{HI}(\phi) \right), \text{ WHERE } V_{HI} = \frac{\lambda^2 (\phi^2 - M^2)^2}{16}
  \]
  is the Potential of φ. Also g is the Determinant Of The Background Metric and We Allow for a Kinetic Mixing Through the Function f_K(φ).

- **At the vacuum, φ = M ≪ 1 so that f_R(⟨φ⟩) ≈ 1 (in Reduced Planck Units With m_P = 1) to Guarantee the Ordinary Einstein Gravity At Low Energy**

- **We can Write S in the Einstein Frame (EF) as follows**
  \[
  S = \int d^4x \sqrt{-\tilde{g}} \left( \frac{1}{2} \tilde{R} + \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} - \tilde{V}_{HI}(\tilde{\phi}) \right)
  \]

- **Performing a Conformal Transformation** according which we define the EF Metric \( \tilde{g}_{\mu\nu} \) and Introduce the EF Canonically Normalized Field, \( \tilde{\phi} \), and Potential, \( \tilde{V} \), Defined As Follows:
  \[
  \tilde{g}_{\mu\nu} = f_R g_{\mu\nu}, \quad \left( \frac{d\tilde{\phi}}{d\phi} \right)^2 = f_R, \quad \frac{f_K}{f_R} + \frac{3}{2} \left( \frac{f_{R,\phi}}{f_R} \right)^2 \text{ AND } \tilde{V}_{HI}(\tilde{\phi}) = \frac{V_{HI}(\phi)}{f_R(\phi)^2}.
  \]

---

The General Framework

Our starting point is the action in the Jordan Frame (JF) of a Higgs field $\phi$ non-minimally coupled to the Ricci scalar curvature, $\mathcal{R}$, through a frame function $f_R(\phi)$. This is:

$$S = \int d^4 x \sqrt{-g} \left( -\frac{1}{2} f_R(\phi) \mathcal{R} + \frac{f_K(\phi)}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V_{HI}(\phi) \right), \quad \text{where} \quad V_{HI} = \lambda^2 (\phi^2 - M^2)^2 / 16$$

is the potential of $\phi$. Also $g$ is the determinant of the background metric and we allow for a kinetic mixing through the function $f_K(\phi)$.

- At the vacuum, $\phi = M \ll 1$ so that $f_R(\langle \phi \rangle) \simeq 1$ (in reduced Planck units with $m_P = 1$) to guarantee the ordinary Einstein gravity at low energy.

- We can write $S$ in the Einstein Frame (EF) as follows:

$$S = \int d^4 x \sqrt{-\tilde{g}} \left( -\frac{1}{2} \tilde{\mathcal{R}} + \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} - \tilde{V}_{HI}(\tilde{\phi}) \right)$$

Performing a conformal transformation\(^1\) according which we define the EF metric $\tilde{g}_{\mu\nu}$ and introduce the EF canonically normalized field, $\tilde{\phi}$, and potential, $\tilde{V}$, defined as follows:

$$\tilde{g}_{\mu\nu} = f_R g_{\mu\nu}, \quad \left( \frac{d\tilde{\phi}}{d\phi} \right)^2 = f^2 = \frac{f_K}{f_R} + \frac{3}{2} \left( \frac{f_{R,\phi}}{f_R} \right)^2 \quad \text{and} \quad \tilde{V}_{HI}(\tilde{\phi}) = \frac{V_{HI}(\tilde{\phi}(\phi))}{f_R(\tilde{\phi}(\phi))^2}.$$

- We observe that $f_R$ affects both $J$ and $\tilde{V}_{HI}$. On the other hand, $f_K$ influences exclusively $J$.

- $J$ (and so $f_K$) has an impact on the inflationary observables.

- The analysis of non-MHI in the EF using the standard slow-roll approximation is equivalent with the analysis in JF.

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Observational Requirements

- The **number of e-foldings**, \( \tilde{N}_* \), that the scale \( k_* = 0.05/\text{Mpc} \) suffers during non-MHI has to be sufficient to resolve the horizon and flatness problems of standard Big Bang:

\[
\tilde{N}_* = \int_{\phi_f}^{\phi_*} d\phi \frac{\tilde{V}_{\text{HI}}}{\tilde{V}_{\text{HI},\phi}} = \int_{\phi_f}^{\phi_*} d\phi J^2 \frac{\tilde{V}_{\text{HI}}}{\tilde{V}_{\text{HI},\phi}} \simeq 61.3 + \ln \frac{\tilde{V}_{\text{HI}}(\phi_*)^{1/2}}{\tilde{V}_{\text{HI}}(\phi_f)^{1/4}} + \frac{1}{2} \ln f_R(\phi_*)
\]

Where \( \phi_* [\tilde{\phi}_*] \) is the value of \( \phi [\tilde{\phi}] \) when \( k_* \) crosses outside the inflationary horizon;

\( \phi_f [\tilde{\phi}_f] \) is the value of \( \phi [\tilde{\phi}] \) at the end of non-MHI which can be found from the condition:

\[
\max\{\tilde{\epsilon}(\phi_f), \tilde{\eta}(\phi_f)\} = 1, \quad \text{with} \quad \tilde{\epsilon} = \frac{1}{2} \left( \frac{\tilde{V}_{\text{HI},\phi}}{\tilde{V}_{\text{HI}}} \right)^2 = \frac{1}{2J^2} \left( \frac{\tilde{V}_{\text{HI},\phi}}{\tilde{V}_{\text{HI}}} \right)^2 \quad \text{AND} \quad \tilde{\eta} = \frac{\tilde{V}_{\text{HI},\phi\phi}}{\tilde{V}_{\text{HI}}} = \frac{1}{J^2} \left( \frac{\tilde{V}_{\text{HI},\phi\phi}}{\tilde{V}_{\text{HI}}} - \frac{\tilde{V}_{\text{HI},\phi}}{\tilde{V}_{\text{HI}}} \frac{\tilde{V}_{\text{HI},\phi}}{\tilde{V}_{\text{HI}}} J_{\phi\phi} \right).
\]

\[\text{Planck Collaboration (2015); Bicep2/Keck Array and Planck Collaborations (2015)}\]
**Observational Requirements**

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  \( \phi_f [\tilde{\phi}_f] \) is the value of \( \phi [\tilde{\phi}] \) at the end of non-MHI which can be found from the condition:

  \[
  \max[\tilde{\epsilon}(\phi_f), \tilde{\eta}(\phi_f)] = 1, \quad \tilde{\epsilon} = \frac{1}{2} \left( \frac{\tilde{V}_{HI,\phi}}{\tilde{V}_{HI}} \right)^2 = \frac{1}{2} J^2 \left( \frac{\tilde{V}_{HI,\phi}}{\tilde{V}_{HI}} \right)^2 \quad \text{AND} \quad \tilde{\eta} = \frac{\tilde{V}_{HI,\phi}}{\tilde{V}_{HI}} = \frac{1}{J^2} \left( \frac{\tilde{V}_{HI,\phi}}{\tilde{V}_{HI}} - \frac{\tilde{V}_{HI,\phi}}{\tilde{V}_{HI}} J, \phi \right).
  \]

- **The Amplitude** \( A_s \) **of the Power Spectrum** of the curvature perturbations is to be consistent with Planck data:
  \[
  A_s^{1/2} = \frac{1}{2 \sqrt{3} \pi} \frac{\tilde{V}_{HI}(\tilde{\phi}_*)^{3/2}}{|\tilde{V}_{HI,\phi}(\tilde{\phi}_*)|} = \frac{|J(\phi_*)|}{2 \sqrt{3} \pi} \frac{\tilde{V}_{HI}(\tilde{\phi}_*)^{3/2}}{|\tilde{V}_{HI,\phi}(\tilde{\phi}_*)|} = 4.627 \times 10^{-5}
  \]

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\(^2\)Planck Collaboration (2015); Bicep2/Keck Array and Planck Collaborations (2015)
The **Number of e-foldings**, $\hat{N}_\star$, that the Scale $k_\star = 0.05$/Mpc Suffers During non-MHI has to be Sufficient to Resolve the Horizon and Flatness Problems of Standard Big Bang:

$$\hat{N}_\star = \int_{\phi_f}^{\phi_\star} d\phi \frac{\hat{V}_{\text{HI}}(\phi)}{\hat{V}_{\text{HI},\phi}} = \int_{\phi_f}^{\phi_\star} d\phi J^2 \frac{\hat{V}_{\text{HI}}(\phi)}{\hat{V}_{\text{HI},\phi}} \approx 61.3 + \ln \frac{\hat{V}_{\text{HI}}(\phi_\star)}{\hat{V}_{\text{HI}}(\phi_f)}^{1/4} + \frac{1}{2} \ln f_R(\phi_\star)$$

Where $\phi_\star [\hat{\phi}_\star]$ is the Value of $\phi [\hat{\phi}]$ When $k_\star$ Crosses Outside The Inflationary Horizon; $\phi_f [\hat{\phi}_f]$ is the Value of $\phi [\hat{\phi}]$ at the end of non-MHI Which Can Be Found From The Condition:

$$\max[\hat{\epsilon}(\phi_f), \eta(\phi_f)] = 1, \quad \text{With} \quad \hat{\epsilon} = \frac{1}{2} \left( \frac{\hat{V}_{\text{HI},\phi}}{\hat{V}_{\text{HI}}} \right)^2 = \frac{1}{2J^2} \left( \frac{\hat{V}_{\text{HI},\phi}}{\hat{V}_{\text{HI}}} \right)^2 \quad \text{AND} \quad \hat{\eta} = \frac{\hat{V}_{\text{HI},\phi\phi}}{\hat{V}_{\text{HI}}} = \frac{1}{J^2} \left( \frac{\hat{V}_{\text{HI},\phi\phi}}{\hat{V}_{\text{HI}}} - \frac{\hat{V}_{\text{HI},\phi}}{\hat{V}_{\text{HI}}} \frac{\hat{J}_\phi}{J} \right).$$

- **The Amplitude** $A_S$ of the Power Spectrum of the Curvature Perturbations is To Be Consistent with *Planck* Data:

$$A_S^{1/2} = \frac{1}{2 \sqrt{3} \pi} \frac{\hat{V}_{\text{HI}}(\hat{\phi}_\star)^{3/2}}{|\hat{V}_{\text{HI},\phi}(\hat{\phi}_\star)|} = \frac{|J(\phi_\star)|}{2 \sqrt{3} \pi} \frac{\hat{V}_{\text{HI}}(\hat{\phi}_\star)^{3/2}}{|\hat{V}_{\text{HI},\phi}(\phi_\star)|} = 4.627 \cdot 10^{-5}$$

- **The (Scalar) Spectral Index**, $n_s$, Its Running, $\alpha_s$, And The Tensor-To-Scalar Ratio $r$ are to be Consistent With the Fitting of the *Planck* Results by the $\Lambda$CDM Model (at 95% c.l.):

$$n_s = 1 - 6\hat{\epsilon}_\star + 2\hat{\eta}_\star = 0.968 \pm 0.0045, \quad |\alpha_s| = |2 \left( 4\hat{\eta}_\star - (n_s - 1)^2 \right) / 3 - 2\hat{\epsilon}_\star | \ll 0.001 \quad \text{AND} \quad r = 16\hat{\epsilon}_\star < 0.07,$$

Where $\hat{\xi} = \hat{\nu}_{\text{HI},\phi} \hat{V}_{\text{HI},\phi\phi\phi}/\hat{V}_{\text{HI}}^2 = \hat{V}_{\text{HI},\phi} \hat{\eta}, /\hat{V}_{\text{HI}} J^2 + 2\hat{\epsilon}_\star$ And The Variables With Subscript $\star$ Are Evaluated at $\phi = \phi_\star$.

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$^2$Planck Collaboration (2015); Bicep2/Keck Array and Planck Collaborations (2015)
Observational Requirements

- **The Number of e-foldings**, \( \tilde{N}_* \), that the Scale \( k_* = 0.05/\text{Mpc} \) Suffers During non-MHI has to be Sufficient to Resolve the Horizon and Flatness Problems of Standard Big Bang:

\[
\tilde{N}_* = \int_{\phi_f}^{\phi_*} d\phi \frac{V_{\text{HI}}}{V_{\text{HI},\phi}} = \int_{\phi_f}^{\phi_*} d\phi \frac{1}{2} J^2 \left( \frac{V_{\text{HI},\phi}}{V_{\text{HI}}} \right)^2 \approx 61.3 + \ln \frac{V_{\text{HI}}(\phi_*)^{1/2}}{V_{\text{HI}}(\phi_f)^{1/4}} + \frac{1}{2} \ln f_R(\phi_*)
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Where \( \phi_* [\tilde{\phi}_*] \) is The Value of \( \phi [\tilde{\phi}] \) When \( k_* \) Crosses Outside The Inflationary Horizon;

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\[
\max[\epsilon(\phi_f), \eta(\phi_f)] = 1, \quad \text{With} \quad \epsilon = \frac{1}{2} \left( \frac{V_{\text{HI},\phi}}{V_{\text{HI}}} \right)^2 = \frac{1}{2} J^2 \left( \frac{V_{\text{HI},\phi}}{V_{\text{HI}}} \right)^2 \quad \text{AND} \quad \eta = \frac{V_{\text{HI},\phi}}{V_{\text{HI}}} = \frac{1}{J^2} \left( \frac{V_{\text{HI},\phi}}{V_{\text{HI}}} - \frac{V_{\text{HI},\phi}}{J} \right).
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- **The Amplitude** \( A_s \) of the Power Spectrum of the Curvature Perturbations is To Be Consistent with Planck Data:

\[
A_s^{1/2} = \frac{1}{2 \sqrt{3} \pi} \left( \frac{V_{\text{HI}}(\phi_*)^{3/2}}{V_{\text{HI},\phi}(\phi_*)} \right) = \frac{|J(\phi_*)|}{2 \sqrt{3} \pi} \left( \frac{V_{\text{HI}}(\phi_*)^{3/2}}{V_{\text{HI},\phi}(\phi_*)} \right) = 4.627 \cdot 10^{-5}
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- The (Scalar) Spectral Index, \( n_s \), Its Running, \( \alpha_s \), and The Tensor-To-Scalar Ratio \( r \) are to be Consistent With the Fitting of the Planck Results by the \( \Lambda \)CDM Model (at 95% c.l.):

\[
n_s = 1 - 6\epsilon_* + 2\tilde{\eta}_* = 0.968 \pm 0.0045, \quad |\alpha_s| = |2(4\tilde{\epsilon}_*^2 - (n_s - 1)^2)/3 - 2\tilde{\xi}_*| \ll 0.001 \quad \text{AND} \quad r = 16\tilde{\epsilon}_* < 0.07,
\]

Where \( \tilde{\xi} = \frac{V_{\text{HI},\phi} V_{\text{HI}}}{V_{\text{HI}}} \), \( \tilde{\eta}_* = \frac{V_{\text{HI},\phi}}{V_{\text{HI}}} \), \( \tilde{\xi}_* = \frac{V_{\text{HI},\phi} V_{\text{HI}}}{V_{\text{HI}}} \), \( J = 2 + 2\tilde{\epsilon} \) AND The Variables With Subscript \( * \) Are Evaluated at \( \phi = \phi_* \).

- The Combined Bicep2/Keck Array and Planck Results\(^2\) Although Do Not Exclude Inflationary Models With Negligible \( r \)'s, They Seem to Favor Those With \( r \)'s of Order 0.01 Since \( r = 0.028^{+0.026}_{-0.025} \Rightarrow 0.003 \lesssim r \lesssim 0.054 \) at 68% c.l.

\(^2\)Planck Collaboration (2015); Bicep2/Keck Array and Planck Collaborations (2015)
The Two Regimes of Pure non-MHI

- non-MHI has been originally formulated as follows:

\[ V_{\text{HI}} = \lambda \phi^4 / 4, \quad \text{with} \quad f_R = 1 + c_R \phi^2 \quad \text{and} \quad f_K = 1. \]
Variants of Non-Minimal Higgs Inflation (non-MHI)

From Pure to Kinetically Modified non-Minimal Higgs Inflation

The Two Regimes of Pure non-MHI

- non-MHI has been originally formulated as follows:

\[ V_{\text{HI}} = \lambda \phi^4 / 4, \text{ with } f_R = 1 + c_R \phi^2 \text{ and } f_K = 1. \]

- The resulting model exhibits the following Two Regimes:

  - The Weak \( c_R \) Regime, with \( c_R \ll 1 \) or \( \phi > 1 \) and \( c_R \)-dependent observables converging towards their values in MHI, i.e., \( n_s \approx 1 - 3 \tilde{N}_* = 0.947 \) and \( r \approx 4n/\tilde{N}_* \approx 0.28 \) for \( n = 4 \) respectively (\( \tilde{N}_* = 60 \)).

  - The Strong \( c_R \) Regime, with \( c_R \gg 1 \) and \( \phi < 1 \) and \( c_R \)-independent observables:

\[ n_s \approx 1 - 2/\tilde{N}_* = 0.965 \text{ and } r \approx 12/\tilde{N}_*^2 = 0.0036. \]
The Two Regimes of Pure non-MHI

- **non-MHI Has Been Originally Formulated As Follows:**
  \[ V_{HI} = \lambda \phi^4 / 4, \quad \text{With} \quad f_R = 1 + c_R \phi^2 \quad \text{and} \quad f_K = 1. \]

- **The Resulting Model Exhibits The Following Two Regimes:**
  - **The Weak \( c_R \) Regime**, with \( c_R \ll 1 \) or \( \phi > 1 \) and \( c_R \)-Dependent Observables Converging Towards Their Values In MHI, i.e., \( n_s \approx 1 - 3\tilde{N}_* = 0.947 \) and \( r \approx 4n/\tilde{N}_* \approx 0.28 \) for \( n = 4 \) Respectively (\( \tilde{N}_* = 60 \)).
  - **The Strong \( c_R \) Regime**, with \( c_R \gg 1 \) and \( \phi < 1 \) and \( c_R \)-Independent Observables:
    \[ n_s \approx 1 - 2/\tilde{N}_* = 0.965 \quad \text{and} \quad r \approx 12/\tilde{N}_*^2 = 0.0036. \]

- **In the Latter, Very Predictive Regime**, The Model Faces Problems With **Perturbative Unitarity**.
From Pure to Kinetically Modified non-Minimal Higgs Inflation

**The Ultraviolet (UV) Cut-off Scale ($\Lambda_{UV}$)**

- In particular, the validity of the Effective Theory implies:

$$ (a) \quad \tilde{V}_{HI}(\phi_*)^{1/4} \leq \Lambda_{UV} \quad \text{for} \quad (b) \quad \phi \leq \Lambda_{UV} $$

Where $\Lambda_{UV}$ is the Ultraviolet Cut-off of the Effective Theory and $\tilde{V}_{HI}(\phi_*)^{1/4}$ is the Inflationary Scale.

- To find $\Lambda_{UV}$, we analyze the Small-Field Behavior of the Theory expanding $S$ about $\langle \phi \rangle \approx 0$ in terms of $\phi$. We have

$$ J^2 = \left( \frac{d\phi}{d\phi} \right)^2 = \frac{f_K}{f_R} + \frac{6c_R^2 \phi^2}{f_R^2} \quad \Rightarrow \quad \langle J \rangle = 1 \quad \text{for} \quad \langle f_K \rangle = 1, \text{i.e.,} \quad \phi = \phi \quad \text{At the Vacuum of the Theory} $$

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From Pure to Kinetically Modified non-Minimal Higgs Inflation

The Ultraviolet (UV) Cut-off Scale ($\Lambda_{UV}$)

- In particular, the validity of the effective theory implies:
  
  $$(a) \quad \bar{V}_{HI}(\phi_*)^{1/4} \leq \Lambda_{UV} \quad \text{for} \quad (b) \quad \phi \leq \Lambda_{UV}$$

  Where $\Lambda_{UV}$ is the Ultraviolet Cut-off of the effective theory and $\bar{V}_{HI}(\phi_*)^{1/4}$ is the inflationary scale.

- To find $\Lambda_{UV}$, we analyze the small-field behavior of the theory expanding $S$ about $\langle \phi \rangle \approx 0$ in terms of $\hat{\phi}$. We have

  $$J^2 = \left( \frac{d\hat{\phi}}{d\phi} \right)^2 = \frac{f_K}{f_R} + \frac{6c_R^2\phi^2}{f_R^2} \implies \langle J \rangle = 1 \quad \text{for} \quad \langle f_K \rangle = 1, \text{i.e.,} \quad \hat{\phi} = \phi \text{ at the vacuum of the theory}$$

- For any $c_R$ we obtain $\Lambda_{UV} = m_P/c_R$ since the expansions about $\langle \phi \rangle = 0$ are $c_R$ dependent:

  $$J^2 \phi^2 = (1 - c_R\phi^2 + 6c_R^2\phi^4 + \cdots)\phi^2 \quad \text{and} \quad \bar{V}_{HI} = \frac{\lambda^2\phi^4}{2} \left( 1 - 2c_R\phi^2 + 3c_R^2\phi^4 - 4c_R^3\phi^6 + \cdots \right)$$

  Since the term which yields the smallest denominator for $c_R > 1$ is $6c_R^2\phi^2$ we find $\Lambda_{UV} = m_P/c_R \ll m_P$.

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THE ULTRAVIOLET (UV) CUT-OFF SCALE ($\Lambda_{UV}$)

- In particular, the validity of the effective theory implies:
  \[
  \begin{align*}
  (a) & \quad \tilde{V}_{HI}(\phi_*)^{1/4} \leq \Lambda_{UV} \quad \text{for (b) } \phi \leq \Lambda_{UV}
  \end{align*}
  \]

Where $\Lambda_{UV}$ is the ultraviolet cut-off of the effective theory and $\tilde{V}_{HI}(\phi_*)^{1/4}$ is the inflationary scale.

- To find $\Lambda_{UV}$, we analyze the small-field behavior of the theory expanding $S$ about $\langle \phi \rangle = 0$ in terms of $\phi$. We have
  \[
  J^2 = \left( \frac{d\phi}{d\phi} \right)^2 = \frac{f_K}{f_R} + \frac{6c_R^2\phi^2}{f_R^2} \quad \Rightarrow \quad \langle J \rangle = 1 \quad \text{for} \quad \langle f_K \rangle = 1, \text{i.e.,} \quad \phi = \phi \quad \text{at the vacuum of the theory}
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  \]

Since the term which yields the smallest denominator for $c_R > 1$ is $6c_R^2\phi^2$ we find $\Lambda_{UV} = m_p/c_R \ll m_p$.

- If we introduce a non-canonical kinetic mixing such that
  \[
  \langle f_K \rangle = c_K \quad \text{and} \quad c_R = r_{RK}c_K
  \]

The expansions above are rewritten in terms of the new parameter $r_{RK}$
  \[
  J^2 \phi^2 = \left( 1 - r_{RK}\phi^2 + 6r_{RK}^2\phi^2 + r_{RK}^2\phi^4 + \cdots \right) \phi^2 \quad \text{and} \quad \tilde{V}_{HI} = \frac{\lambda^2\phi^4}{2c_K^2} \left( 1 - 2r_{RK}\phi^2 + 3r_{RK}^2\phi^4 - 4r_{RK}^3\phi^6 + \cdots \right).
  \]

Consequently, no problem with the perturbative unitarity emerges for $r_{RK} \leq 1$, even if $c_R$ and $c_K$ are large.

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From Pure to Kinetically Modified non-Minimal Higgs Inflation

The Ultraviolet (UV) Cut-off Scale ($\Lambda_{\text{UV}}$)

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  \[ (a) \quad \tilde{V}_{\text{HI}}(\phi_*)^{1/4} \leq \Lambda_{\text{UV}} \quad \text{for} \quad (b) \quad \phi \leq \Lambda_{\text{UV}} \]

Where $\Lambda_{\text{UV}}$ is the ultraviolet cut-off of the effective theory and $\tilde{V}_{\text{HI}}(\phi_*)^{1/4}$ is the inflationary scale.

- To find $\Lambda_{\text{UV}}$, we analyze the small-field behavior of the theory expanding $S$ about $\langle \phi \rangle \approx 0$ in terms of $\tilde{\phi}$. We have

  \[ J^2 = \left( \frac{d\tilde{\phi}}{d\phi} \right)^2 = \frac{f_K}{f_R} + \frac{6c_R^2\phi^2}{f_R^2} \quad \Rightarrow \quad \langle J \rangle = 1 \quad \text{for} \quad \langle f_K \rangle = 1, \text{i.e.,} \quad \tilde{\phi} = \phi \text{ at the vacuum of the theory} \]

- For any $c_R$ we obtain $\Lambda_{\text{UV}} = m_P/c_R$ since the expansions about $\langle \phi \rangle = 0$ are $c_R$ dependent:

  \[ J^2 \phi^2 = \left( 1 - c_R\phi^2 + 6c_R^2\phi^2 + c_R^2\phi^4 + \cdots \right) \phi^2 \quad \text{and} \quad \tilde{V}_{\text{HI}} = \frac{\lambda^2\phi^4}{2} \left( 1 - 2c_R\phi^2 + 3c_R^2\phi^4 - 4c_R^3\phi^6 + \cdots \right). \]

Since the term which yields the smallest denominator for $c_R > 1$ is $6c_R^2\phi^2$ we find $\Lambda_{\text{UV}} = m_P/c_R \ll m_P$.

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the expansions above are rewritten in terms of the new parameter $r_{RK}$.

  \[ J^2 \phi^2 = \left( 1 - r_{RK}\phi^2 + 6r_{RK}^2\phi^2 + r_{RK}^2\phi^4 + \cdots \right) \phi^2 \quad \text{and} \quad \tilde{V}_{\text{HI}} = \frac{\lambda^2\phi^4}{2c_K^2} \left( 1 - 2r_{RK}\phi^2 + 3r_{RK}^2\phi^4 - 4r_{RK}^3\phi^6 + \cdots \right). \]

Consequently, no problem with the perturbative unitarity emerges for $r_{RK} \leq 1$, even if $c_R$ and $c_K$ are large.

We propose to analyze models of kinetically modified non-MHI with $f_K = c_Kf_R^m$ where $c_K = (c_R/r_{RK})$ & $r_{RK} \leq 1$

(Note that $\langle f_K \rangle = c_K$ & $\langle f_R \rangle = 1$ and the extra functional uncertainty in $f_K$ is parameterized by $f_R^m$)

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**Analytical Results**

- **Kinetically Modified** non-MHI is defined in the JF with the following ingredients

\[ V_{HI} = \lambda^2 (\phi^2 - M^2)^2 / 16, \quad \text{with} \quad f_R = 1 + c_R \phi^2 \quad \text{and} \quad f_K = c_K f_R^m. \]
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- The slow-roll parameters are determined using the standard formulae in the EF:

\[ \bar{\epsilon} = \frac{8}{\phi^2 c_K f_R^{1+m}} \quad \text{and} \quad \bar{\eta} = \left( \frac{3}{2} - \left( 1 + (1 + m)c_R \phi^2 / 2 \right) \right) \bar{\epsilon}. \]

- The number of e-foldings is calculated to be

\[ \bar{N} \approx \frac{c_K \phi^2}{2n} \frac{f_R^{1+m} - 1}{(1 + m)c_R \phi^2} \Rightarrow \phi_\star \approx \sqrt{\frac{f_{m \star} - 1}{c_K}}, \]

where \( f_{m \star} = \left( 1 + 8(m + 1)r_{RK} \bar{N}_\star \right)^{1/(1+m)} \).

- For every \( m \), there is a lower bound on \( c_K \), above which \( \phi_\star < 1 \). Indeed, \( \phi_\star < 1 \Rightarrow c_K \geq (f_{m \star} - 1)/r_{RK} \).
**Analytical Results**

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\[ \tilde{e} = 8(\phi^2 c_K f_R^{1+m}) \quad \text{AND} \quad \tilde{\eta} = \left( 3/2 - \left( 1 + (1 + m)c_R \phi^2 / 2 \right) \right)\tilde{e}. \]

- The **Number of e-foldings Is Calculated to be**

\[ \tilde{N}_* \approx \frac{c_K \phi_*^2}{2n} \frac{f_R^{1+m} - 1}{(1 + m) c_R \phi_*^2} \quad \Rightarrow \quad \phi_* \approx \sqrt{\frac{f_R^{1+m} - 1}{c_K}}, \]

where \( f_R^{1+m} = (1 + 8(m + 1)r_{RK}\tilde{N}_*)^{1/(1+m)}. \)

- For every \( m \), there is a **Lower Bound on \( c_K \)**, above which \( \phi_* < 1 \). Indeed, \( \phi_* < 1 \Rightarrow c_K \geq (f_R^{1+m} - 1) / r_{RK}. \)

- The **Normalization of \( A_s \) Implies A Dependence of \( \lambda \) on \( c_K \) for Every \( r_{RK} \)**, i.e. \( \lambda = 16 \sqrt{3A_s \pi c_K r_{RK}^{3/2} / (f_R^{1+m} - 1)^{3/2} f_R^{(1+m)/2}}. \)

- A **Clear Efficient Dependence of The Observables On \( r_{RK} \)** Arises. Indeed,

\[ n_s = 1 - (f_R^{1+m} - 1) \frac{m - 1 + (m + 2)f_R^{1+m}}{(f_R^{1+m} - 1)f_R^{1+m}(1 + m)\tilde{N}_*}, \quad r = \frac{16(f_R^{1+m} - 1)}{(f_R^{1+m} - 1)f_R^{1+m}(1 + m)\tilde{N}_*}, \quad \alpha_s = \alpha_s \left( f_R^{1+m}, \tilde{N}_*, r_{RK} \right). \]

- E.g., Expanding the Relevant Formulas for \( 1 / \tilde{N}_* \ll 1 \) We Find For \( m = 1 \):

\[ n_s \approx 1 - 3/2 \tilde{N}_* - 3/8(\tilde{N}_* r_{RK})^{1/2}, \quad r \approx 1/2\tilde{N}_* r_{RK} + 2/(\tilde{N}_* r_{RK})^{1/2}, \quad \alpha_s \approx -3/2\tilde{N}_* - 9/16(\tilde{N}_* r_{RK})^{1/2}. \]
Numerical Results

- The free parameters of the models, for fixed $m$, are $r_{RK}$ and $\lambda/c_K$ (and not $c_K$, $c_R$ and $\lambda$). Since if we perform a rescaling $\phi = \tilde{\phi}/\sqrt{c_K}$ then $f_K = f_R^m$, with $f_R = 1 + r_{RK}\tilde{\phi}^{n/2}$ and $V_{HI} \approx \lambda^2\tilde{\phi}^4/16c_K^2$ for $M < 1$. 

Numerical Results

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- Imposing the Planck constraints for \( \bar{N}_* = 55 \) we obtain the following allowed curves:
**Numerical Results**

- The free parameters of the models, for fixed $m$, are $r_{\text{RK}}$ and $\lambda/c_\text{K}$ (and not $c_\text{K}$, $c_\text{R}$ and $\lambda$). Since if we perform a rescaling $\phi = \tilde{\phi}/\sqrt{c_\text{K}}$ then $f_\text{K} = f_\text{R}^m$, with $f_\text{R} = 1 + r_{\text{RK}}\tilde{\phi}^{n/2}$ and $V_{\text{HI}} \approx \lambda^2 \phi^4/16c_\text{K}^2$. For $M < 1$.

- Imposing the Planck constraints for $\hat{N}_* = 55$ we obtain the following allowed curves:

- For $m = 0$ we reveal the results of the original non-MHI although with $\phi < 1$. 

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**C. Pallis**

**Gravitational Waves & Leptogenesis From Higgs Inflation in SUGRA**
**Numerical Results**

- **The Free Parameters Of The Models, For Fixed m, are** $r_{RK}$ **and** $\lambda/c_K$ **(and not** $c_K$, $c_R$ **and** $\lambda$). **Since**
- **If We Perform A Rescaling** $\phi = \tilde{\phi}/\sqrt{c_K}$ **Then** $f_K = f_R^m$, **With** $f_R = 1 + r_{RK} \tilde{\phi}^{n/2}$ **and** $V_{HI} \approx \lambda^2 \tilde{\phi}^4/16c_K^2$ **For** $M < 1$.
- **Imposing The Planck Constraints for** $\tilde{N}_* = 55$ **we obtain the Following Allowed Curves:**

![Graph showing allowed curves for $r_{002}$ and $n_s$](image)

- **For** $m = 0$ **we reveal the results of the original non-MHI although with** $\phi < 1$.
- **For** $m > 0$ **the curves move to the right of the line for** $m = 0$ **and fill the 1-σ observationally favored ranges for quite natural** $r_{RK}$’s — e.g. $0.0048 \lesssim r_{RK} \lesssim 0.5$ **for** $m = 1$.
- **In SUGRA Realizations Of The Models the Positivity Of** $\kappa_-$ **Provides An Upper Bound on** $r_{RK}$ **which is Translated to a Lower Bound on** $r$, **more restrictive than that arising from** $r_{RK} \leq 1$.  

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**Inflation Analysis**

Variants of Non-Minimal Higgs Inflation (non-MHI)

<table>
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C. Pallis

Gravitational Waves & Leptogenesis From Higgs Inflation in SUGRA

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Numerical Results

- The free parameters of the models, for fixed $m$, are $r_{R K}$ and $\lambda/c_K$ (and not $c_K$, $c_R$ and $\lambda$). Since if we perform a rescaling $\phi = \tilde{\phi}/\sqrt{c_K}$ then $f_K = f_R^m$, with $f_R = 1 + r_{R K} \tilde{\phi}^{2n/2}$ and $V_{H I} \approx \lambda^2 \tilde{\phi}^4/16c_K^2$ for $M < 1$.

- Imposing the Planck constraints for $N_\star = 55$ we obtain the following allowed curves:

For $m = 0$ we reveal the results of the original non-MHI although with $\phi < 1$.

For $m > 0$ the curves move to the right of the line for $m = 0$ and fill the 1-$\sigma$ observationally favored ranges for quite natural $r_{R K}$’s – e.g. $0.0048 \lesssim r_{R K} \lesssim 0.5$ for $m = 1$.

In SUGRA realizations of the models the positivity of $\kappa_-$ provides an upper bound on $r_{R K}$ which is translated to a lower bound on $r$, more restrictive than that arising from $r_{R K} \lesssim 1$.

For $m = 1$, $n_s = 0.968$ entails $r_{R K} = 0.015$ which corresponds to $r = 0.043$. Lying within the 65% c.l allowed margin.

The achieved $r$’s are possibly detectable in the next generation experiments is expected to achieve a precision for $r$ of the order of $10^{-3}$. E.g., Core+, LiteBird, Bicep3/it Keck Array.
### Numerical Results

- **The Free Parameters Of The Models, For Fixed m**, are \( r_{RK} \) and \( \lambda/c_K \) (and not \( c_K \), \( c_R \) and \( \lambda \)). Since

  If We Perform A Rescaling \( \phi = \bar{\phi}/\sqrt{c_K} \) Then \( f_K = f_R^m \), With \( f_R = 1 + r_{RK}\bar{\phi}^{n/2} \) and \( V_{HI} \approx \lambda^2\bar{\phi}^4/16c_K^2 \) For \( M < 1. \)

- **Imposing the Planck Constraints for \( \bar{N}_* = 55 \) we obtain the following allowed curves:**

  ![Graph showing allowed regions for \( r_{RK} \) and \( n_s \) for different values of \( m \).]

  - For \( m = 0 \) we reveal the results of the original non-MHI Although With \( \phi < 1. \)
  - For \( m > 0 \) the curves move to the right of the line for \( m = 0 \) and fill the 1-\( \sigma \) observationally favored ranges for quite natural \( r_{RK} \)'s – e.g. \( 0.0048 \leq r_{RK} \leq 0.5 \) for \( m = 1. \)
  - In SUGRA Realizations Of The Models the Positivity of \( \kappa \) Provides an Upper Bound on \( r_{RK} \) Which is Translated to a Lower Bound on \( r \), More Restrictive Than That Arising from \( r_{RK} \leq 1. \)
  - For \( m = 1 \), \( n_s = 0.968 \) entails \( r_{RK} = 0.015 \) Which Corresponds to \( r = 0.043 \). Lying Within The 65% c.l Allowed Margin.
  - The Achieved \( r \)'s are Possibly Detectable in the Next Generation Experiments is Expected To Achieve A Precision For \( r \) of the Order of \( 10^{-3} \). E.g., Core+, LiteBird, Bicep3/it Keck Array.
  - Repeating the Same Analysis For \( (-1) \leq m \leq 10 \) We Obtain \( 0.2 \leq m \leq 4 \) & \( 0.0029 \leq r \leq 0.07 \) For \( n_s = 0.968. \)
THE (SEMI)LOGARITHMIC KÄHLER POTENTIAL

THE GENERAL FRAMEWORK

- **The General EF Action For The Scalar Fields** $z^\alpha$ **Plus Gravity In Four Dimensional, $N = 1$ SUGRA is:**

$$S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} \hat{R} + K_{\alpha\beta} \tilde{g}^{\mu\nu} D_\mu z^\alpha D_\nu z^{\star \beta} - \tilde{V} \right)$$

Where $\tilde{V} = \tilde{V}_F + \tilde{V}_D$,

$K$ is the Kähler potential with $K_{\alpha\beta} = K_{\zeta\lambda} z^\zeta z^{\star \lambda} = \frac{\partial^2 K}{\partial z^\alpha \partial z^{\star \beta}} > 0$ and $k^{\alpha}_{\beta} K_{\alpha\gamma} = \delta_{\gamma}^{\beta}$; $D_\mu z^\alpha = \partial_\mu z^\alpha - A_\mu k^\alpha_A$.

($A_\mu^A$: the Vector Gauge Fields and $k^\alpha_A$: the Killing Vector, Defining The Gauge Transformations Of The Scalars.)

$$\tilde{V}_F = e^K \left( K_{\alpha\beta} F_\alpha F^{\star \beta} - 3|W|^2 \right) \quad \text{with} \quad F_\alpha = W_{\zeta\alpha} z^\zeta + K_{\zeta\alpha} W; \quad \tilde{V}_D = \frac{1}{2} g^2 D_\alpha D_\alpha \quad \text{with} \quad D_\alpha = z_{\alpha} (T_\alpha)^A_B K_{A\beta}.$$

**Therefore, Implementing non-MHI Within SUGRA Requires The Appropriate Selection Of $W$ and $K$**

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The (Semi)Logarithmic Kähler Potential

The General Framework

- The General EF Action For The Scalar Fields $z^\alpha$ Plus Gravity In Four Dimensional, $N = 1$ SUGRA is:

$$S = \int d^4 x \sqrt{-g} \left( -\frac{1}{2} \mathcal{R} + K_{\alpha\bar{\beta}} \tilde{g}^{\mu\nu} D_\mu z^\alpha D_\nu z^{\bar{\beta}} - \hat{V} \right) \text{ where } \hat{V} = \hat{V}_F + \hat{V}_D,$$

where $K$ is the Kähler potential with $K_{\alpha\bar{\beta}} = K_{\bar{\alpha}z^{\bar{\beta}}} = \frac{\partial^2 K}{\partial z^{\alpha} \partial z^{\bar{\beta}}} > 0$ and $K^{\bar{\beta}\alpha} K_{\alpha\bar{\gamma}} = \delta^{\bar{\beta}}_{\bar{\gamma}}$; $D_\mu z^\alpha = \partial_\mu z^\alpha - A_\mu A^\alpha_{\bar{\alpha}}$; $\hat{V}_F = e^K \left( K^{\alpha\bar{\beta}} F_\alpha F_{\bar{\beta}} - 3|W|^2 \right)$ with $F_\alpha = W_{\bar{\alpha}} + K_{\bar{\alpha}z} W$; $\hat{V}_D = \frac{1}{2} g^2 D_a D_a$ with $D_a = z_\alpha (T_a)^\alpha_{\bar{\beta}} z^{\bar{\beta}}$.

Therefore, implementing non-MHI within SUGRA requires the appropriate selection of $W$ and $K$.

- If we set $K = -N \ln \left( -\frac{\Omega}{N} \right)$ and perform a conformal transformation, $S$ in JF Reads

$$S = \int d^4 x \sqrt{-g} \left( \frac{\Omega}{2N} \mathcal{R} + \left( \Omega_{\bar{z}^{\alpha}z^{\bar{\beta}}} + \frac{3 - N}{N} \frac{\Omega_{\bar{z}^{\alpha}}}{} \frac{\Omega_{\bar{z}^{\bar{\beta}}}}{\Omega} \right) D_\mu z^\alpha D^\mu z^{\bar{\beta}} - \frac{27}{N^3} \Omega A_\mu A^\mu - V \right), \text{ } \Omega: \text{ Frame Function}$$

The General Framework

- The General EF Action for the Scalar Fields $z^\alpha$ Plus Gravity in Four Dimensional, $N = 1$ SUGRA is:

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where $\bar{V} = \bar{V}_F + \bar{V}_D$,

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$$\bar{V}_F = e^K \left( K^{\alpha\beta} F_\alpha F^*_\beta - 3|W|^2 \right) \text{ with } F_\alpha = W_{z^\alpha} + K_{z^\alpha} W; \quad \bar{V}_D = \frac{1}{2} g^2 D_a D_a \text{ with } D_a = z^\alpha (T_a)^\alpha_{\beta} K_{z^\beta}.$$

Therefore, implementing non-MHI within SUGRA requires the appropriate selection of $W$ and $K$.

- If we set $4 K = -N \ln \left( -\Omega/N \right)$ and perform a conformal transformation, $S$ in JF reads

$$S = \int d^4x \sqrt{-\bar{g}} \left( \frac{\Omega}{2N} \mathcal{R} + \left( \frac{\Omega}{N} \frac{\Omega z^\alpha}{\Omega} \right) \frac{3 - N}{N} \frac{D_\mu z^\alpha D^\mu z^\beta}{\Omega} - \frac{27}{N^2} \Omega A_\mu A^\mu - V \right), \quad \Omega: \text{ Frame Function}$$

- We observe that $\Omega$ enters the kinetic terms of the $z^\alpha$'s too. $S$ can exhibit non-minimal couplings of $z^\alpha$'s to $\mathcal{R}$ if

  - $A_\mu = 0$ where $A_\mu = -i N \left( D_\mu z^\alpha \Omega_\alpha - D_\mu z^{\*\alpha} \Omega_{\*\alpha} \right) / 6 \Omega$. This happens when $\text{arg} z^\alpha = 0$ or $z^\alpha = 0$ during inflation;

  - We can decompose $\Omega$ to an holomorphic $\Omega_H = \Omega_H(z^\alpha)$ and a kinetic (real) $\Omega_K = \Omega_K(z^\alpha z^{*\alpha})$ part, with $\Omega_H \gg \Omega_K \simeq \delta_{\alpha\beta} z^\alpha z^{*\beta}$ where we restrict ourselves to the lowest order quadratic terms. Therefore

$$\Omega = \Omega_K - N \left( \Omega_H(z^\alpha) + \Omega_H^{*}(z^{*\alpha}) \right) \Rightarrow K = -N \ln \left( \Omega_H(z^\alpha) + \Omega_H^{*}(z^{*\alpha}) - \Omega_K/N \right).$$

Although $N = 3$ is standard since it assures canonical terms for $z_\alpha$'s, $0 < N \neq 3$ is totally acceptable.

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C. Pallis

Gravitational Waves & Leptogenesis From Higgs Inflation in SUGRA
Selecting Conveniently the Super- and Kähler Potential Potential

- **We Use 3 Superfields** $z^1 = \Phi$, $z^2 = \bar{\Phi}$, **Charged Under a Local Symmetry**, e.g. $U(1)_{B-L}$, and $z^3 = S$ ("Stabilizer" Field).
- **$W$ is Uniquely Determined Using** $U(1)_{B-L}$ **and $R$ Symmetries.**

$$W = \lambda S \left( \Phi \bar{\Phi} - M^2/4 \right) \implies \langle S \rangle = 0, |\langle \Phi \rangle| = |\langle \bar{\Phi} \rangle| = M_{BL}/2,$$

Since in the SUSY limit we get

$$V_{HI} \sim \lambda^2 |\Phi \bar{\Phi} - M^2/4|^2 + \lambda^2 |S|^2 (|\Phi|^2 + |\bar{\Phi}|^2) + D - \text{terms}$$

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5 C.P. and N. Toumbas (2016).
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- **$W$ is uniquely determined using $U(1)_{B-L}$ and $R$ symmetries.**

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Since in the SUSY limit we get $V_{HI} \sim \lambda^2 |\Phi \bar{\Phi} - M^2/4|^2 + \lambda^2 |S|^2 (|\Phi|^2 + |\bar{\Phi}|^2) + D - terms$

- **If we set** $S = 0$, **the only surviving term** of $\tilde{V}$ is

$$\tilde{V}_{HI} = e^K K \bar{S} S^* |W_S|^2 = \lambda^2 K \bar{S} S^* / f_R^n \quad \text{where} \quad f_R = \Omega / N, \quad \text{and} \quad K = -N \ln f_R.$$  

\[5\text{ C.P. and N. Toumbas (2016).} \]
Softly Broken Shift Symmetry For Higgs Fields

Selecting Conveniently the Super- and Kähler Potential Potential

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\[
\tilde{V}_{HI} = e^{K} K^{SS^*} |W_S|^2 = \lambda^2 K^{SS^*} / f_R \quad \text{where} \quad f_R = -\Omega/N, \quad \text{and} \quad K = -N \ln f_R. \\
\]

- **Kinetically Modified non-MHI** could be obtained selecting the following Kähler Potential invariant under \( U(1)_{B-L} \) and \( R \):

\[
\tilde{K}_1 = -2 \ln (1 + c_+ F_+ - (1 + c_+ F_+)^m c_- F_-/2) + K_S \quad \text{or} \quad \tilde{K}_2 = -2 \ln (1 + c_+ F_+) + (1 + c_+ F_+)^m c_- F_- + K_S. \\
\]

Given that non-MHI takes place along the path with \( \Phi = \bar{\Phi}^* \) we can convince ourselves that \( 1 + c_+ F_+ = f_R \) and \( F_- \) assists us to obtain \( f_K = c_K f_R^m \) where \( c_K = c_- \), \( c_R = c_+ \) and \( r_{RK} = r_\pm = c_+/c_- \).

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\*C.P. and N. Toumbas (2016).
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- **W Is Uniquely Determined** Using $U(1)_{B-L}$ and $R$ Symmetries.

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Since in the SUSY limit we get
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V_{HI} \sim \lambda^2 \left| \Phi \bar{\Phi} - M^2/4 \right|^2 + \lambda^2 |S|^2 (|\Phi|^2 + |\bar{\Phi}|^2) + D - \text{terms}
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\[
\tilde{V}_{HI} = e^{K} K^S S^* \left| W_S \right|^2 = \lambda^2 K^S S^* / f_R^N \quad \text{WHERE} \quad f_R = -\Omega / N, \quad \text{AND} \quad K = -N \ln f_R.
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\[
\tilde{K}_1 = -2 \ln \left( 1 + c_+ F_+ - (1 + c_+ F_+)^m c_- F_- / 2 \right) + K_S \quad \text{OR} \quad \tilde{K}_2 = -2 \ln \left( 1 + c_+ F_+ + (1 + c_+ F_+)^m - c_- F_- + K_S \right).
\]

Given that non-MHI takes place along the path with $\Phi = \bar{\Phi}^*$ we can convince ourselves that $1 + c_+ F_+ = f_R$ and $F_-$ assists us to obtain $f_K = c_K f_R^m$ **where** $c_K = c_-, c_R = c_+$ **and** $r_{RK} = r_{\pm} = c_+/c_-$.  

- **Stabilization of the $S = 0$ Direction** can be achieved without invoking higher order terms, if we select $S U(2)/U(1)$.

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\[5\] C.P. and N. Toumbas (2016).
Selecting Conveniently the Super- and Kähler Potential Potential

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- \( W \) is uniquely determined using \( U(1)_{B-L} \) and \( R \) symmetries.

\[
W = \lambda S \left( \Phi \Phi - M^2/4 \right) \rightarrow \langle S \rangle = 0, |\langle \Phi \rangle| = |\langle \bar{\Phi} \rangle| = M_{BL}/2,
\]

Since in the SUSY limit we get
\[
V_{\text{HI}} \sim \lambda^2 |\Phi \bar{\Phi} - M^2/4|^2 + \lambda^2 |S|^2 (|\Phi|^2 + |\bar{\Phi}|^2) + D \text{ – terms}
\]

- **If we set** \( S = 0 \), the **only surviving term** of \( \tilde{V} \) is

\[
\tilde{V}_{\text{HI}} = e^K K^{SS^*} \left| W_S \right|^2 = \lambda^2 K^{SS^*} / f^R \] where \( f^R = -\Omega/N \), and \( K = -N \ln f^R \).

- **Kinetically Modified non-MHI** could be obtained selecting the following Kähler potential invariant under \( U(1)_{B-L} \) and \( R \):

\[
\tilde{K}_1 = -2 \ln (1 + c_+ F_+ - (1 + c_+ F_+)^m c_- F_- / 2) + K_S \quad \text{or} \quad \tilde{K}_2 = -2 \ln (1 + c_+ F_+) + (1 + c_+ F_+)^{-m-1} c_- F_- + K_S
\]

Given that non-MHI takes place along the path with \( \Phi = \bar{\Phi}^* \) we can convince ourselves that \( 1 + c_+ F_+ = f^R \) and \( F_- \) assists us to obtain \( f_K = c_K f^R \) where \( c_K = c_- \), \( c_R = c_+ \) and \( r_{RK} = r_{\pm} = c_+ / c_- \).

- **Stabilization of the** \( S = 0 \) **Direction** can be achieved without invoking higher order terms, if we select \(^5\):

\[
K_S = N_S \ln \left( 1 + |S|^2 / N_S \right), \text{ which parameterizes the compact manifold } SU(2)/U(1).
\]

- For \( c_+ \ll c_- \), our models are completely **natural**, because the theory enjoys the following enhanced symmetries:

\[
\bar{\Phi} \rightarrow \bar{\Phi} + c^*, \ \Phi \rightarrow \Phi + c \ (c \in \mathbb{C}) \quad \text{and} \quad \frac{S}{\sqrt{N_S}} \rightarrow \frac{aS}{\sqrt{N_S}} + b + \frac{-b^* S}{\sqrt{N_S} + a^*} \text{ with } |a|^2 + |b|^2 = 1 \text{ in the limits } c_+ \to 0 \text{ & } \lambda \to 0
\]

\(^5\) C.P. and N. Toumbas (2016).
Selecting Conveniently the Super- and Kähler Potential Potential

- We use 3 superfields \( z^1 = \Phi, z^2 = \bar{\Phi}, \) charged under a local symmetry, e.g., \( U(1)_{B-L}, \) and \( z^3 = S \) ("stabilizer" field).
- \( W \) is uniquely determined using \( U(1)_{B-L} \) and \( R \) symmetries.

\[
W = \lambda S (\Phi \Phi - M^2/4) \quad \implies \quad \langle S \rangle = 0, \quad |\langle \Phi \rangle| = |\langle \bar{\Phi} \rangle| = M_{BL}/2,
\]

Since in the SUSY limit we get

\[
V_{HI} \sim \lambda^2 \Phi \bar{\Phi} - M^2/4 \| S \|^2 (|\Phi|^2 + |\bar{\Phi}|^2) + D - \text{terms}
\]

- If we set \( S = 0 \), the only surviving term of \( \tilde{V} \) is

\[
\tilde{V}_{HI} = e^K K^{S \bar{S}} \left| W_S \right|^2 = \lambda^2 K^{S \bar{S}}/f_R^N \quad \text{where} \quad f_R = -\Omega/N, \quad \text{and} \quad K = -N \ln f_R.
\]

- Kinetically modified non-MHI could be obtained selecting the following Kähler potential invariant under \( U(1)_{B-L} \) and \( R \):

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\[
\bar{\Phi} \to \Phi + c^*, \quad \Phi \to \Phi + c \quad (c \in \mathbb{C}) \quad \text{and} \quad S/\sqrt{N_S} \to \frac{aS}{\sqrt{N_S}} + b \quad \text{with} \quad |a|^2 + |b|^2 = 1 \quad \text{in the limits} \quad c_+ \to 0 \quad \text{and} \quad \lambda \to 0
\]

- For \( m = 0 \) [\( m = 1 \)], \( F_+ \) and \( F_- \) in \( \tilde{K}_1 [\tilde{K}_2] \) are totally decoupled, i.e., no higher order term is needed.

\[5\] C.P. and N. Toumbas (2016).

C. Pallis

Gravitational Waves & Leptogenesis From Higgs Inflation in SUGRA
A $B - L$ Extension of MSSM

Promoting to local the already existing $U(1)_{B-L}$ global symmetry of the MSSM, we obtain:

---

6 G. Dvali, G. Lazarides and Q. Shafi (1999).
A \( B - L \) Extension of MSSM

Promoting to local the already existing \( U(1)_{B-L} \) global symmetry of the MSSM, we obtain:

- A Convenient Candidate for Higgs-Inflaton. Since a GUT scale phase transition can be implemented via

\[
W_{\text{HI}} = \lambda S (\Phi \Phi - M^2/4), \quad \text{which can also support non-MHI.}
\]

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\(^6\) G. Dvali, G. Lazarides and Q. Shafi (1999).
Beyond MSSM With Several Consequences

A $B - L$ Extension of MSSM

Promoting to local the already existing $U(1)_{B-L}$ global symmetry of the MSSM, we obtain:

- A Convenient Candidate for Higgs-Inflaton. Since a GUT scale phase transition can be implemented via

$$W_{HI} = \lambda S (\Phi\Phi - M^2/4),$$

which can also support non-MHI.

- Generation of Masses for the Light Neutrinos. Through the Type I Seesaw Mechanism which can be realized by the terms

$$W_{RHN} = \lambda_{ij} c \bar{\Phi} v_i^c v_j^c + h_{ij} v_i^c L_j H_u.$$

Note that the three RHs, $v_i^c$, are necessary to cancel the $B - L$ gauge anomaly.

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Note that the Three RHNs, $v_i^c$, Are Necessary To Cancel the $B – L$ Gauge Anomaly.

- **A Motivation for the Origin of the $\mu$ Term.** This can be Explained If We Combine $W_{HI}$ With
  
  $$W_{\mu} = \lambda_\mu S H_u H_d. (I)$$

The Part Of The Scalar Potential Which Includes The Soft Susy Breaking Terms Corresponding to $W_{HI} + W_{\mu}$

$$V_{\text{soft}} = \left(\lambda A_\lambda S \Phi \bar{\Phi} + \lambda_\mu A_\mu S H_u H_d - a_S S \lambda M^2 + \text{h.c.} \right) + m_\alpha^2 |z^\alpha|^2 \quad \text{with } z^\alpha = \Phi, \bar{\Phi}, S, H_u, H_d,$$

where $m_\alpha, A_\lambda, A_\mu$ and $a_S$ are Soft Susy Breaking Mass Parameters. Minimizing $V_{\text{tot}} = V_{\text{SUSY}} + V_{\text{soft}}$ and Substituting in $V_{\text{soft}}$ the SUSY v.e.vs of $\Phi$ and $\bar{\Phi}$ we get

$$\langle V_{\text{tot}}(S) \rangle = 2\lambda^2 M^2 S^2 - \lambda (|A_\lambda| + |a_S|) M^2 S,$$

where $m_S \ll M$

The Minimized $\langle V_{\text{tot}}(S) \rangle$ w.r.t $S$ leads to a non Vanishing $\langle S \rangle$ as follows:

$$\partial \langle V_{\text{tot}}(S) \rangle / \partial S = 0 \quad \Rightarrow \quad \langle S \rangle \approx (|A_\lambda| + |a_S|) / 2\lambda.$$

Therefore, the Generated $\mu$ Parameter From Eq. (I) is $\mu = \lambda_\mu \langle S \rangle \approx \lambda_\mu (|A_\lambda| + |a_S|) / 2\lambda \sim \lambda_\mu m_{3/2} / \lambda$. Successful non-MHI Needs $\lambda_\mu \leq 9 \cdot 10^{-6}$ and $\lambda \geq 6.6 \cdot 10^{-4}$ ($r = 0.03$). Therefore, $\mu \geq 1 \text{ TeV}$ Implies $m_{3/2} \geq 75 \text{ TeV}$.

---

Note: 6 G. Dvali, G. Lazarides and Q. Shafi (1999).
**The Relevant Super- & Kähler Potentials**

- **We Focus on a Superpotential invariant under the** $G_{SM} \times U(1)_{B-L}$ **Gauge Group:**

  $$W = \lambda S \left( \bar{\Phi} \Phi - M^2 \right)$$

  **to Achieve non-MHI & Break** $U(1)_{B-L}$

  + $\lambda_{\mu S} H_u H_d$

  **to Generate** $\mu = \lambda_{\mu m_3/2}/\lambda \sim 1$ TeV

  + $\lambda_{ij} \bar{\Phi} v_i^c v_j^c$

  **to Generate Majorana Masses for Neutrinos**

  & **Ensure The Inflaton Decay**

  + $h_{vi j} v_i^c L_j H_u$

  **to Generate Dirac Masses for Neutrinos**

  + $W$ **of MSSM with** $\mu = 0$

---

<table>
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<th>Representations under $G_{SM} \times U(1)_{B-L}$</th>
<th>Global Symmetries</th>
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<tr>
<td>$e_i^c$</td>
<td>$(1, 1, 1, -1)$</td>
<td>$R$</td>
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<tr>
<td>$\nu_i^c$</td>
<td>$(1, 1, 1, -1)$</td>
<td>$B$</td>
</tr>
<tr>
<td>$l_i$</td>
<td>$(1, 1, 2, 1)$</td>
<td>$L$</td>
</tr>
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<td>$u_i^c$</td>
<td>$(3, 2, 1, 1/3)$</td>
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<td>$(3, 2, 1, 1/3)$</td>
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<td>$(\bar{3}, 2, -1/3)$</td>
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**Matter Fields**

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<td>$\bar{\Phi}$</td>
</tr>
<tr>
<td>$\Phi$</td>
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The Relevant Super- & Kähler Potentials

- We focus on a superpotential invariant under the $G_{SM} \times U(1)_{B-L}$ gauge group:

$$W = \lambda S \left( \bar{\Phi} \Phi - M^2 \right)$$

To achieve non-MHI & break $U(1)_{B-L}$

+ $\lambda_{\mu} S H_u H_d$

To generate $\mu = \lambda_{\mu} m_3/\lambda \sim 1$ TeV

+ $\lambda_{ij} v_i^c v_j^c$

To generate majorana masses for neutrinos & ensure the inflaton decay

+ $h_{ij} v_i^c L_j H_u$

To generate dirac masses for neutrinos

+ $W$ of MSSM with $\mu = 0$

<table>
<thead>
<tr>
<th>SUPER-FIELDS</th>
<th>REPRESENTATIONS UNDER $G_{SM} \times U(1)_{B-L}$</th>
<th>GLOBAL SYMMETRIES</th>
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<tr>
<td>$e_i^c$</td>
<td>$(1, 1, 1, -1)$</td>
<td>0</td>
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<tr>
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<td>0</td>
</tr>
<tr>
<td>$l_i$</td>
<td>$(1, 1, 2, 1)$</td>
<td>2</td>
</tr>
<tr>
<td>$u_i^c$</td>
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<td>1</td>
</tr>
</tbody>
</table>

Matter Fields

Higgs Fields

| $H_d$          | $(1, 2, -1/2, 0)$                             | 0 | 0 | 0 |
| $H_u$          | $(1, 2, 1/2, 0)$                             | 0 | 0 | 0 |
| $S$            | $(1, 1, 0, 0)$                               | 4 | 0 | 0 |
| $\bar{\Phi}$  | $(1, 1, 0, 2)$                               | 0 | 0 | -2 |
| $\Phi$         | $(1, 1, 0, -2)$                              | 0 | 0 | 2 |

- The above $W$ may cooperate with the following Kähler potential potentials which respect the imposed symmetries

$$K_1 = -2 \ln \left( 1 + c_+ F_+ - (1 + c_+ F_+)^m c_- F_- / 2 \right) + N_S \ln \left( 1 + \sum_X |X|^2 / N_S \right) \quad \text{(where } F_{\pm} = |\Phi \pm \Phi^*|^2 \text{)}$$

$$K_2 = -2 \ln \left( 1 + c_+ F_+ \right) + (1 + c_+ F_+)^{m-1} c_- F_- + N_S \ln \left( 1 + \sum_X |X|^2 / N_S \right) ,$$

$$K_3 = -2 \ln \left( 1 + c_+ F_+ \right) + N_S \ln \left( 1 + \sum_X |X|^2 / N_S + (1 + c_+ F_+)^{m-1} c_- F_- / N_S \right) , \quad \text{where } X = S, H_u, H_d, v_i^c .$$

Placing $\sum_X |X|^2$ outside the argument of $\ln$, we obtain tighter restrictions on $\lambda_{\mu}$ – see below.
The Inflationary Potential

- If we use the parametrization:
  \[ \Phi = \phi e^{i\theta} \cos \theta_\Phi / \sqrt{2} \quad \text{and} \quad \bar{\Phi} = \phi e^{i\bar{\theta}} \sin \theta_\Phi / \sqrt{2} \quad \text{with} \quad 0 \leq \theta_\Phi \leq \pi / 2 \quad \text{and} \quad X^\beta = (X_1^\beta + iX_2^\beta) / \sqrt{2} \]

Where \( X^\beta = S, H_u, H_d, \nu_i^c \). We can show that a D-flat direction is

\[ \theta = \bar{\theta} = 0, \quad \theta_\Phi = \pi / 4 \quad \text{and} \quad X^\beta = 0 \quad (\text{: I}) \]
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- The only surviving term of \( \tilde{V}_F \) along the path in Eq. (I) is (independent of \( c_- \) & \( m \))

\[ \tilde{V}_{HI} = e^K K^{S*S^*} |W_S|^2 = \frac{\lambda^2 (\phi^2 - M^2)^2}{16 f_R^2}, \quad \text{where} \quad f_R = 1 + c_+ \phi^2 \quad \text{plays the role of a non-minimal coupling} \]
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  \[ \hat{V}_{HI} = e^K K^{SS^*} |W_S|^2 = \frac{\lambda^2 (\phi^2 - M^2)^2}{16 f_R^2}, \quad \text{where} \quad f_R = 1 + c_+ \phi^2 \text{ plays the role of a non-minimal coupling} \]

- Along the path in Eq. (I) the Kähler metric \( K_{\alpha\beta} \) takes the form
  \[ (K_{\alpha\beta}) = \text{diag} \left( M_K, K_{\bar{\beta}\bar{\beta}} \right) \quad \text{with} \quad M_K = \frac{1}{f_R^2} \begin{pmatrix} \kappa & \bar{\kappa} \\ \bar{\kappa} & \kappa \end{pmatrix}, \quad \begin{cases} \kappa = c_- f_R^{1+m} - 2c_+ \\ \bar{\kappa} = 2c_+^2 \phi^2 \end{cases} \quad \text{and} \quad K_{\bar{\beta}\bar{\beta}} = 1 \]

- \( M_K \) can be diagonalized via a similarity transformation as follows:
  \[ U_K M_K U_K^T = \text{diag} (\kappa_+, \kappa_-), \quad \text{where} \quad U_K = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \quad \text{and} \quad \begin{cases} \kappa_+ = c_- \left( f_R^{1+m} + 2r_\pm (c_+ \phi^2 - 1) \right) / f_R^2, \\ \kappa_- = c_- \left( f_R^m - 2r_\pm \right) / f_R > 0 \quad \Rightarrow \quad r_\pm < 1/2 \end{cases} \]
The Inflationary Scenario

The Inflationary Potential

- If we use the parametrization:
  \[ \Phi = \phi e^{i\theta} \cos \theta_\Phi / \sqrt{2} \quad \text{and} \quad \tilde{\Phi} = \phi e^{i\tilde{\theta}} \sin \theta_\Phi / \sqrt{2} \quad \text{with} \quad 0 \leq \theta_\Phi \leq \pi/2 \quad \text{and} \quad X^\beta = \left( X_1^\beta + iX_2^\beta \right) / \sqrt{2} \]

  Where \( X^\beta = S, H_u, H_d, \nu^c \). We can show that a D-flat direction is

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- \( M_K \) can be diagonalized via a similarity transformation as follows:

  \[ U_K M_K U_K^\top = \text{diag} \left( \kappa_+, \kappa_- \right), \quad \text{where} \quad U_K = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \quad \text{and} \quad \left\{ \begin{align*} \kappa_+ &= c_- \left( f_R^{1+m} + 2r_\pm (c_+ \phi^2 - 1) \right) / f_R^2, \\ \kappa_- &= c_- \left( f_R^m - 2r_\pm \right) / f_R > 0 \Rightarrow r_\pm < 1/2 \end{align*} \right. \]

- The EF canonically normalized fields, which are denoted by hat, can be obtained as follows:

  \[ \frac{d\tilde{\phi}}{d\phi} = J = \sqrt{\kappa_+}, \quad \tilde{\theta}_+ = \frac{J \phi \theta_+}{\sqrt{2}}, \quad \tilde{\theta}_- = \sqrt{\frac{\kappa_-}{2}} \phi \theta_- \quad \text{and} \quad \tilde{\theta}_\Phi = \phi \sqrt{\kappa_-} \left( \theta_\Phi - \frac{\pi}{4} \right), \quad \left( \tilde{x}_1^\beta, \tilde{x}_2^\beta \right) = \left( x_1^\beta, x_2^\beta \right) \]

- We can check the stability of the trajectory in Eq. (I) w.r.t. the fluctuations of the various fields, i.e.

  \[ \frac{\partial V}{\partial \tilde{z}^{\alpha\beta}} \bigg|_{\text{Eq. (I)}} = 0 \quad \text{and} \quad \tilde{m}^2_{\tilde{z}^{\alpha\beta}} > 0 \quad \text{where} \quad \tilde{m}^2_{\tilde{z}^{\alpha\beta}} = \text{Egv} \left[ \tilde{M}_{\alpha\beta}^2 \right] \quad \text{with} \quad \tilde{M}_{\alpha\beta}^2 = \frac{\partial^2 V}{\partial \tilde{z}^{\alpha\beta} \partial \tilde{z}^{\alpha\beta}} \bigg|_{\text{Eq. (I)}} \quad \text{and} \quad z^{\alpha} = \theta_- \phi, \theta_+ \phi, x_1^\beta, x_2^\beta. \]
The Inflationary Scenario

**Stability and Radiative Corrections**

The mass spectrum along the inflationary trajectory

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<td></td>
<td></td>
<td>$K = K_1$</td>
</tr>
<tr>
<td>2 Real Scalars</td>
<td>$\tilde{\theta}_+$</td>
<td>$\tilde{m}^2_{\theta_+}$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{\theta}_\Phi$</td>
<td>$\tilde{m}^2_{\theta_\Phi}$</td>
</tr>
<tr>
<td>1 Complex Scalars</td>
<td>$\tilde{s}, \tilde{\bar{s}}$</td>
<td>$\tilde{m}^2_{s}$</td>
</tr>
<tr>
<td>4 Complex Scalars</td>
<td>$H_\pm$</td>
<td>$\tilde{m}^2_{H_\pm}$</td>
</tr>
<tr>
<td>3 Complex Scalars</td>
<td>$\nu^c_i$</td>
<td>$\tilde{m}^2_{\nu^c_i}$</td>
</tr>
<tr>
<td>1 Gauge boson</td>
<td>$A_{BL}$</td>
<td>$M^2_{BL}$</td>
</tr>
<tr>
<td>4 Weyl Spinors</td>
<td>$\tilde{\psi}_\pm$</td>
<td>$\tilde{m}^2_{\psi_\pm}$</td>
</tr>
<tr>
<td></td>
<td>$\psi_{i\nu^c}$</td>
<td>$\tilde{m}^2_{\psi_{i\nu^c}}$</td>
</tr>
<tr>
<td></td>
<td>$\lambda_{BL}, \tilde{\psi}_{\Phi-}$</td>
<td>$M^2_{BL}$</td>
</tr>
</tbody>
</table>

- We can obtain $\forall \alpha, \tilde{m}_c^2 > 0$. Especially
  \[
  \tilde{m}_s^2 > 0 \iff N_S < 6 \quad \text{and} \quad \tilde{m}_{H-}^2 > 0 \iff \lambda_\mu \leq \lambda (1 + 1/N_S) \phi / 4 \quad \text{(E.G.} \quad \lambda_\mu < 9 \cdot 10^{-6} \quad \text{for} \quad r_\pm = 0.03).\]
- We can obtain $\forall \alpha, \tilde{m}_c^2 > \tilde{H}_HI^2$ and so any inflationary perturbations of the fields other than $\phi$ are safely eliminated;
- $M_{BL} \neq 0$ signals the fact that $U(1)_{B-L}$ is broken and so, no Topological Defects are produced.
- The one-loop radiative corrections à la Coleman-Weinberg to $\tilde{V}_{HI}$ can be kept under control provided that
  - $M_{BL}^2 > m_P^2$ and $\tilde{m}_{\theta_\Phi}^2 > m_P^2$ are not taken into account.
- The renormalization group mass scale $\Lambda$ is determined by requiring $\Delta \tilde{V}_{HI}(\phi_\star) = 0$ or $\Delta \tilde{V}_{HI}(\phi_f) = 0$.
Perturbative Reheating

- At the SUSY vacuum, the Inflaton and the RHNS, $\nu^c_i$, acquire masses $\tilde{m}_{\delta \phi}$ and $M_{\nu^c}$ respectively given by

$$\tilde{m}_{\delta \phi} \simeq \frac{\lambda M_{BL}}{\sqrt{2(1 - 2r_\pm)\langle J \rangle}} \quad \text{(E.g. } 9 \cdot 10^{10} \text{ GeV for } r_\pm = 0.03) \quad \text{and} \quad M_{\nu^c} = 2 \lambda_{\nu^c} M_{BL},$$

where we restore $m_P$ in the formulas. The mass $\tilde{m}_{\delta \phi}$ is only $r_\pm$ dependent and increases with it.
Perturbative Reheating

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where we restore \( m_\nu \) in the formulas. The mass \( \tilde{m}_{\delta \phi} \) is only \( r_\pm \) dependent and increases with it.

- The inflaton can decay perturbatively into:

  - A pair of RHNS \( (\tilde{\nu}_i) \) with Majorana masses \( M_{i \nu \nu} \) through the following decay width

\[
\Gamma_{\delta \phi \to \nu_i^c} = \frac{\lambda_{i \nu \nu}^2}{16\pi} \tilde{m}_{\delta \phi} \left(1 - \frac{4M_{i \nu \nu}^2}{\tilde{m}_{\delta \phi}}\right)^{3/2} \quad \text{with} \quad \lambda_{i \nu \nu} = \frac{\tilde{m}_{\delta \phi}}{2\langle J \rangle M} \left(1 - 3c_+ \frac{M^2}{m_\nu^2}\right) \quad \text{arising from} \quad L_{\delta \phi \to \nu_i^c} = \lambda_{i \nu \nu} \tilde{\phi} \nu_i^c \nu_i^c.
\]

  - \( H_u \) and \( H_d \) through the following decay width

\[
\Gamma_{\delta \phi \to H} = \frac{2}{8\pi} \lambda_H^2 \tilde{m}_{\delta \phi} \quad \text{with} \quad \lambda_H = \frac{\lambda_H}{\sqrt{2}} \left(1 - 2c_+ \frac{M^2}{m_\nu^2}\right) \quad \text{arising from} \quad L_{\delta \phi \to H_u H_d} = -\lambda_H \tilde{m}_{\delta \phi} \tilde{\phi} H_u^* H_d^*.
\]

  - MSSM (s)-Particles \( XYZ \) through the following \( c_+ \)-dependent 3-body decay width

\[
\Gamma_{\delta \phi \to XYZ} = \frac{\lambda_y^2}{512\pi^3} \frac{14n_\text{f}}{m_\nu^2} \tilde{m}_{\delta \phi} \quad \text{with} \quad \lambda_y = 2y_3 c_+ \frac{M_{BL}}{\langle J \rangle m_\nu} \quad \text{and} \quad y_3 = h_{t,b,\tau}(\tilde{m}_{\delta \phi}) \approx 0.5.
\]

This decay arises from \( L_{\delta \phi \to XYZ} = -\lambda_y(\tilde{\phi}/m_\nu)(X\psi_Y\psi_Z + Y\psi_X\psi_Z + Z\psi_X\psi_Y) + \text{h.c.} \).
Inflaton Decay & non-Thermal Leptogenesis

Perturbative Reheating

- At the SUSY vacuum, the inflaton and the RHns, $\tilde{\nu}_i^c$, acquire masses $\tilde{m}_{\delta \phi}$ and $M_{\tilde{\nu}_i^c}$ respectively given by
  \[ \tilde{m}_{\delta \phi} \simeq \frac{\lambda M_{BL}}{\sqrt{2(1 - 2r_\pm)}} \] (E.g. $9 \cdot 10^{10}$ GeV for $r_\pm = 0.03$) and $M_{\tilde{\nu}_i^c} = 2\lambda_{\tilde{\nu}_i^c} M_{BL}$.

  Where we restore $m_p$ in the formulas. The mass $\tilde{m}_{\delta \phi}$ is only $r_\pm$ dependent and increases with it.

- The inflaton can decay perturbatively into:
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    \[ \tilde{\Gamma}_{\delta \phi \rightarrow \nu_i^c} = \frac{\lambda_{\tilde{\nu}_i^c}^2}{16\pi} \tilde{m}_{\delta \phi} \left( 1 - \frac{4M_{\tilde{\nu}_i^c}^2}{\tilde{m}_{\delta \phi}^2} \right)^{3/2} \] with $\lambda_{\tilde{\nu}_i^c} = \frac{M_{\tilde{\nu}_i^c}}{2\langle J \rangle M} \left( 1 - 3c_+ \frac{M^2}{m_p^2} \right)$ arising from $L_{\delta \phi \rightarrow \nu_i^c} = \lambda_{\tilde{\nu}_i^c} \tilde{\nu}_i^c \nu_i^c \nu_i^c$.
  - $H_u$ and $H_d$ through the following decay width
    \[ \tilde{\Gamma}_{\delta \phi \rightarrow H_u H_d} = \frac{2}{8\pi} \lambda_H^2 \tilde{m}_{\delta \phi} \] with $\lambda_H = \frac{\lambda_H}{\sqrt{2}} \left( 1 - 2c_+ \frac{M^2}{m_p^2} \right)$ arising from $L_{\delta \phi \rightarrow H_u H_d} = -\lambda_H \tilde{m}_{\delta \phi} \delta \phi H_u^* H_d^*$.
  - MSSM (s)-Particles $XYZ$ through the following $c_+$-dependent 3-body decay width
    \[ \tilde{\Gamma}_{\delta \phi \rightarrow XYZ} = \lambda_y \frac{14n_f}{512\pi^3} \tilde{m}_{\delta \phi} \] with $\lambda_y = 2y_3c_+ \frac{M_{BL}}{\langle J \rangle m_p}$ and $y_3 = h_{t,b,\tau}(\tilde{m}_{\delta \phi}) \approx 0.5$.

  This decay arises from $L_{\delta \phi \rightarrow XYZ} = -\lambda_y(\tilde{\phi}/m_p)(X\psi Y\psi Z + Y\psi X\psi Z + Z\psi X\psi Y) + \text{h.c.}$

- The reheating temperature, $T_{rh}$, is given by
  \[ T_{rh} = \left( \frac{72}{5\pi^2} g_* \right)^{1/4} \tilde{\Gamma}_{\delta \phi}^{1/2} m_{\tilde{p}}^{1/2} \] with $\tilde{\Gamma}_{\delta \phi} = \tilde{\Gamma}_{\delta \phi \rightarrow \nu_i^c} + \tilde{\Gamma}_{\delta \phi \rightarrow H} + \tilde{\Gamma}_{\delta \phi \rightarrow XYZ}$, with $g_* \approx 228.75$.  

C. Pallis
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**Leptogenesis and $\tilde{G}$ Abundance**

- **The Out-of-equilibrium Decay of $\nu_i^c$ can generate an L asymmetry which can be converted to the $B$ yield:**

\[
Y_B = -0.35 \frac{5}{4} \frac{T_{\text{rh}}}{\overline{m}_{\phi}} \frac{\Gamma_{\phi \to \nu_i^c}}{\Gamma_{\phi}} \epsilon_i \quad \text{Here} \quad \overline{m}_\phi < 2M_{\tilde{\nu}^c} \quad \text{For some } i \text{ with } i = 1, 2, 3.
\]

---

### Leptogenesis and $\tilde{G}$ Abundance

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- **The Thermally Produced $\tilde{G}$ Yield At The Onset of BBN Is Estimated To Be:**

$$Y_{\tilde{G}} \approx 1.9 \cdot 10^{-22} T_{\text{th}} / \text{GeV}.$$ 

---

**Leptogenesis and $\tilde{G}$ Abundance**

- **The Out-Of-Equilibrium Decay of $\nu_i^c$ can generate an $L$ asymmetry which can be converted to the $B$ Yield:**
  
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**Post-Inflationary Requirements**

1. **UV Behavior.** As anticipated $\Lambda_{\text{UV}} = 1$ for $r_\pm \leq 1$ since the expansions about $\langle \phi \rangle$ are $r_\pm$ (and not $c_-$ or $c_+$) dependent:

   $$m^2 \phi^2 \approx \left(1 + (m - 1)r_\pm \phi^2 + 6r_\pm^2 \phi^2 + \left(1 - \frac{1}{2} m(m - 3)\right)r_\pm^2 \phi^2 + \cdots \right) \phi^2 \quad \text{and} \quad \tilde{V} \approx \frac{\lambda^2 \phi^4}{16c_-^2} \left(1 - 2r_\pm \phi^2 + 3r_\pm^2 \phi^4 - \cdots \right).$$

---

Inflaton Decay & Non-Thermal Leptogenesis

Leptogenesis and $\tilde{G}$ Abundance

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Here $m_{\delta} < 2 M_{\tilde{\nu}_i}$ For Some $i$ with $i = 1, 2, 3$.

- The Thermally Produced $G$ Yield At The Onset of BBN Is Estimated To Be: $Y_G \approx 1.9 \cdot 10^{-22} T_{th} / \text{GeV}$.

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$$J^2 \phi^2 \approx \left( 1 + (m - 1) r_\pm \phi^2 + 6 r^2 \phi^2 + \left( 1 - \frac{1}{2} m (m - 3) \right) r^2 \phi^2 - \cdots \right) \phi^2$$

And

$$\tilde{V} \approx \frac{\lambda^2 \phi^4}{16 c_-^2} \left( 1 - 2 r_\pm \phi^2 + 3 r^2 \phi^4 - \cdots \right).$$

(ii) Gauge Unification. Although $U(1)_{B-L}$ Gauge Symmetry Does Not Disturb This Gauge Coupling Unification Within MSSM We Determine $M$ Demanding That The Unification Scale $M_{\text{GUT}} \approx 2/2.433 \times 10^{-2}$ is identified with $M_{BL}$ at the Vacuum, i.e.

$$\sqrt{c_- (\langle f_R \rangle - 2 r_\pm) g M} / \sqrt{\langle f_R \rangle} = M_{\text{GUT}} \Rightarrow M \approx M_{\text{GUT}} / g \sqrt{c_- (1 - 2 r_\pm)} \sim 10^{15} \text{ GeV} \text{ with } g \approx 0.7 \text{ (GUT Gauge Coupling).}$$


**Inflaton Decay & non-Thermal Leptogenesis**

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  Here $m_{\delta\phi} < 2M_{\tilde{\nu}^c}$ for some $i$ with $i = 1, 2, 3$.

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and

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(a) $M_{\tilde{\nu}^c} \gtrsim 10 T_{\text{th}}$, (b) $m_{\delta\phi} \gtrsim 2M_{\tilde{\nu}^c}$ and (c) $M_{\tilde{\nu}^c} \lesssim 7.1 M \iff \lambda_{\nu^c} \lesssim 3.5$.

---

LEPTOGENESIS AND $\tilde{G}$ ABUNDANCE

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(iv) The achievement of baryogenesis via non-thermal leptogenesis dictates at 95% c.l. $Y_B = \left( 8.64_{-0.16}^{+0.15} \right) \cdot 10^{-11}$.

---

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POST-INFLATIONARY REQUIREMENTS

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(iv) The Achievement of Baryogenesis via non-thermal leptogenesis dictates at 95% c.l. $Y_B = \left(8.64^{+0.15}_{-0.16}\right) \cdot 10^{-11}$.

(v) $\tilde{G}$ Constraints. Assuming unstable $\tilde{G}$, we impose an upper bound on $Y_{\tilde{G}}$ in order to avoid problems with the SBB nucleosynthesis:

$$Y_{\tilde{G}} \lesssim \begin{cases} 10^{-14} & \Rightarrow T_{\text{th}} \lesssim \begin{cases} 5.3 \cdot 10^7 \text{GeV} \\
5.3 \cdot 10^8 \text{GeV} \end{cases}, \text{for} \quad \tilde{G} \text{ mass } m_{\tilde{G}} \approx \begin{cases} 0.69 \text{ TeV} \\
10.6 \text{ TeV}. \end{cases} \end{cases}$$

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Lepton-Number Asymmetry and Light Neutrino Data

- $m_{iD}$ are the Dirac masses in a basis (called $\nu_i^c$-basis) where $\nu_i^c$ are mass eigenstates. In the weak (primed) basis

\[
U^\dagger m_D U^\dagger = d_D = \text{diag}(m_{1D}, m_{2D}, m_{3D}) \quad \text{where} \quad L' = LU \quad \text{and} \quad \nu^{c'} = U^c \nu^c \quad (: 1).
\]
Lepton-Number Asymmetry and Light Neutrino Data

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  \[ U^\dagger m_D U^c = d_D = \text{diag}(m_{1D}, m_{2D}, m_{3D}) \]  where $L' = LU$ and $\nu^{c'} = U^c \nu^c$ (: 1).

- Working in the $\nu^c_i$-basis, the Type I seesaw formula reads
  \[ m_\nu = -m_D d_{\nu c}^{-1} m_D^T, \]  where $d_{\nu c} = \text{diag}(M_{1\nu}, M_{2\nu}, M_{3\nu})$ with $M_{1\nu} \leq M_{2\nu} \leq M_{3\nu}$ real and positive.

- Replacing $m_D$ from Eq. (1) in the above equation and we extract the mass matrix of light neutrinos in the weak basis
  \[ m_\nu = U^\dagger m_\nu U^* = -d_D U^c d_{\nu c}^{-1} U^c U^\dagger d_D, \]

Which can be diagonalized by the is the unitary PMNS matrix $U_\nu$ parameterized as follows:

\[ U_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot \begin{pmatrix} e^{-i\varphi_1/2} & e^{-i\varphi_2/2} & 1 \end{pmatrix}, \]

with $c_{ij} := \cos \theta_{ij}$, $s_{ij} := \sin \theta_{ij}$, $\delta$ the CP-violating Dirac phase and $\varphi_1$ and $\varphi_2$ the two CP-violating Majorana phases.
LEPTON-NUMBER ASYMMETRY AND LIGHT NEUTRINO DATA

- \( m_{iD} \) are the Dirac masses in a basis (called \( \nu_i^c \)-basis) where \( \nu_i^c \) are mass eigenstates. In the weak (primed) basis
  \[
  U^\dagger m_D U^{c\dagger} = d_D = \text{diag}(m_{1D}, m_{2D}, m_{3D}) \quad \text{where} \quad L' = LU \quad \text{and} \quad \nu^{c'} = U^c \nu^c \quad (\because 1).
  \]
- Working in the \( \nu_i^c \)-basis, the type I seesaw formula reads
  \[
  m_\nu = -m_D d^{-1}_{\nu c} m_D^T, \quad \text{where} \quad d_{\nu c} = \text{diag}(M_{1\nu}, M_{2\nu}, M_{3\nu}) \quad \text{with} \quad M_{1\nu} \leq M_{2\nu} \leq M_{3\nu} \quad \text{real and positive.}
  \]
- Replacing \( m_D \) from Eq. (I) in the above equation and we extract the mass matrix of light neutrinos in the weak basis
  \[
  m_\nu = U^\dagger m_\nu U^* = -d_D U^c d^{-1}_{\nu c} U^{c\dagger} d_D,
  \]

Which can be diagonalized by the is the unitary PMNS matrix \( U_\nu \) parameterized as follows:

\[
U_\nu = \begin{pmatrix}
  c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\
  -c_{23} s_{12} - s_{23} c_{12} s_{13} e^{i \delta} & c_{23} c_{12} - s_{23} s_{12} s_{13} e^{i \delta} & s_{23} c_{13} \\
  s_{23} s_{12} - c_{23} c_{12} s_{13} e^{i \delta} & -s_{23} c_{12} - c_{23} s_{12} s_{13} e^{i \delta} & c_{23} c_{13}
\end{pmatrix} \cdot \begin{pmatrix}
  e^{-i \varphi_1 / 2} & & \\
  & e^{-i \varphi_2 / 2} & \\
  & & 1
\end{pmatrix},
\]

with \( c_{ij} := \cos \theta_{ij} \), \( s_{ij} := \sin \theta_{ij} \), \( \delta \) the CP-violating Dirac phase and \( \varphi_1 \) and \( \varphi_2 \) the two CP-violating Majorana phases.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Best Fit ±1σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta m_{21}^2/10^{-3} \text{eV}^2 )</td>
<td>( 7.6^{+0.19}_{-0.18} )</td>
</tr>
<tr>
<td>( \Delta m_{31}^2/10^{-3} \text{eV}^2 )</td>
<td>( 2.38^{+0.05}_{-0.07} )</td>
</tr>
<tr>
<td>( \sin^2 \theta_{12} / 0.1 )</td>
<td>3.23 ± 0.16</td>
</tr>
<tr>
<td>( \sin^2 \theta_{13} / 0.01 )</td>
<td>2.26 ± 0.12</td>
</tr>
<tr>
<td>( \sin^2 \theta_{23} / 0.1 )</td>
<td>( 5.67^{+0.32}_{-1.24} )</td>
</tr>
<tr>
<td>( \delta / \pi )</td>
<td>1.41^{+0.55}_{-0.4}</td>
</tr>
</tbody>
</table>

- The masses, \( m_\nu \), of \( \nu_i \) are calculated as follows:
  \[
  m_{2\nu} = \sqrt{m_{1\nu}^2 + \Delta m_{21}^2} \quad \text{and}
  \]
  \[
  \begin{cases}
    m_{3\nu} = \sqrt{m_{1\nu}^2 + \Delta m_{31}^2}, & \text{for no } m_\nu \text{'s} \\
    \nu_{1\nu} = \sqrt{m_{3\nu}^2 + |\Delta m_{31}|}, & \text{for IO } m_\nu \text{'s}
  \end{cases}
  \]

- \( \sum_i m_{i\nu} = 0.23 \text{ eV at 95\% c.l. from Planck Data.} \)
**RESULTS**

- To verify the compatibility of the post-inflationary constraints, we focus on the following central values of the inflationary model:
  
  \[(r_\pm, c_-) = (0.03, 146) \rightarrow (n_s, r) = (0.969, 0.03) \& (\lambda, \overline{m}_{\delta\phi}) \simeq (6.6 \cdot 10^{-4}, 10^{11} \text{ GeV})\].

- All the requirements can be met along the lines presented in the \(\lambda_\mu - m_{1D}\) plane.

---

**Table:**

<table>
<thead>
<tr>
<th>Cases</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hierarchy</td>
<td>NO</td>
<td>NO</td>
<td>IO</td>
</tr>
<tr>
<td>(m_r / \text{eV})</td>
<td>0.01</td>
<td>0.05</td>
<td>0.007</td>
</tr>
<tr>
<td>(\Sigma m_r / \text{eV})</td>
<td>0.074</td>
<td>0.17</td>
<td>0.106</td>
</tr>
<tr>
<td>(m_{2D} / \text{GeV})</td>
<td>3</td>
<td>5.5</td>
<td>0.8</td>
</tr>
<tr>
<td>(m_{3D} / \text{GeV})</td>
<td>100</td>
<td>135</td>
<td>10</td>
</tr>
<tr>
<td>(\varphi_1)</td>
<td>-(\pi)</td>
<td>(\pi)</td>
<td>-(\pi)</td>
</tr>
<tr>
<td>(\varphi_2)</td>
<td>0</td>
<td>(\pi / 3)</td>
<td>0</td>
</tr>
<tr>
<td>(M_{1r} / 10^{10} \text{ GeV})</td>
<td>0.9 - 1.7</td>
<td>1 - 1.95</td>
<td>2.6</td>
</tr>
<tr>
<td>(M_{2r} / 10^{11} \text{ GeV})</td>
<td>2.4</td>
<td>5.5</td>
<td>5</td>
</tr>
<tr>
<td>(M_{3r} / 10^{13} \text{ GeV})</td>
<td>20</td>
<td>17</td>
<td>0.64</td>
</tr>
</tbody>
</table>

- We take \(m_{rv} = m_{1\nu}\) for NO \(\nu_i\)'s and \(m_{rv} = m_{3\nu}\) for IO \(\nu_i\)'s.

- In all cases, the inflaton decays exclusively into the lightest of RHNS since \(2M_{i2} > \overline{m}_{\delta\phi}\) for \(i = 2\) and 3.

- \(Y_B\) is equal to its central value and the \(\overline{G}\) constraint is under control for \(m_{3/2} \geq 75\) TeV since we obtain
  
  \[0.9 \leq Y_G / 10^{-14} \leq 5 \quad \text{and} \quad 0.5 \leq T_{rh} / 10^8 \text{GeV} \leq 2.6\].

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C. Pallis

Gravitational Waves & Leptogenesis From Higgs Inflation in SUGRA
**Conclusions**

- **We proposed a variant of non-MHI which can safely accommodate r’s of order 0.01 with subplanckian inflaton values and without causing any problem with the validity of the effective theory.**
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• This scenario can be elegantly implemented within a $B - L$ SUSY GUT, adopting a superpotential determined by a $R$-symmetry and several semi-logarithmic Kähler potentials which respect a softly broken shift-symmetry.
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This scenario can be elegantly implemented within a \( B-L \) SUSY GUT, adopting a superpotential determined by a \( R \)-symmetry and several semi-logarithmic Kähler potentials which respect a softly broken shift-symmetry.

Combined restrictions from baryogenesis via nTL, \( \tilde{G} \) constraints and neutrino data can be met when \( m_{3/2} \geq 75 \text{ TeV} \), \( \lambda \lesssim 7 \cdot 10^{-3} \), and the inflaton decays to \( \nu_1^c \), with \( M_\tilde{\nu^c} \) in the range \( (10^{10} - 10^{14}) \text{ GeV} \).

Thank You!