THE GEOMETRY OF QUANTUM HALL EFFECT: AN EFFECTIVE ACTION FOR ALL DIMENSIONS

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(2+1) DIM QHE

- (2+1) dim systems of (nonrelativistic) electrons in strong magnetic field
- Hall conductivity is quantized

$$\begin{aligned} J^i &= \sigma_H \epsilon^{ij} E_j \\ \sigma_H &= \nu \frac{e^2}{h} \end{aligned}$$

  where $\nu =$ filling fraction

  $\nu = 1, 2, \cdots$ for IQHE and $\nu = 1/3, 1/5, \cdots$ for FQHE.

- Framework for interesting ideas
  - topological field theories (Chern-Simons effective actions)
  - bulk-edge dynamics
  - non-commutative geometries, fuzzy spaces

BASIC FEATURES OF (2+1) DIM IQHE

Charged particle moving on 2d plane (or S^2) in strong external magnetic field (Landau problem)

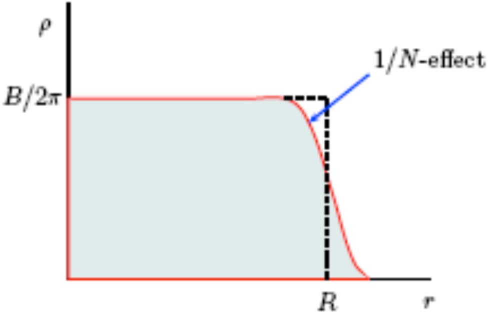
- Distinct Landau levels, separated by energy gap ($\sim\; B$)
- Each Landau level is degenerate
- Lowest Landau level (LLL) :

$$\psi_n \sim z^n e^{-|z|^2/2}$$

$$z = x + iy$$

Many-body problem $\Longrightarrow$ quantum Hall droplets

- Degeneracy of each LL is lifted by confining potential (for example: $V = \frac{1}{2}ur^2$)
- Exclusion principle → N-body ground state = incompressible droplet
- Large $N \to$ sharp boundary



Low energy excitations of droplets $\Longleftrightarrow$ area preserving boundary fluctuations (edge excitations)

Edge dynamics is collectively described by 1d chiral boson ϕ (WEN, STONE,..)

$$S_{\text{edge}} = \int_{\partial D} \Big(\partial_t \phi + u\, \partial_\theta \phi\Big) \partial_\theta \phi, \qquad u \sim \frac{\partial V}{\partial r^2}\Big]_{\text{boundary}}$$

ELECTROMAGNETIC FLUCTUATIONS

In the presence of electromagnetic interactions

$$A_\mu = \underset{\text{Constant } B}{a_\mu} + \underset{\text{Perturbation}}{\delta A_\mu}$$

- The bulk dynamics is described by an effective action

$$S_{\text{bulk}} = S_{CS} = \frac{\nu}{4\pi} \int_D \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

S_{CS} is not gauge invariant in presence of boundaries.

- The edge dynamics is described by

$$S_{\text{edge}} \sim \text{gauged chiral action}$$

Anomaly cancellation between bulk and edge actions,

$$\delta S_{\text{bulk}} + \delta S_{\text{edge}} = 0$$

- The effective action S_{CS} captures the response of the system to electromagnetic fluctuations

$$\begin{aligned} \frac{\delta S_{CS}}{\delta A_0} &= \rho = \frac{\nu}{2\pi} \\ \frac{\delta S_{CS}}{\delta A_i} &= J^i = \frac{\nu}{2\pi}\epsilon^{ij}E_j \end{aligned}$$

Coefficient in front of S_{CS} is related to Hall conductivity.

- What about other transport coefficients?
  - How does the system respond to stress and strain?
  - Calculate stress tensor $\Longleftrightarrow$ couple theory to gravity

EFFECTIVE ACTION IN BACKGROUND GAUGE FIELD AND METRIC

$$\begin{aligned} S[\psi,\psi^\dagger;A_\mu,g_{ij}] &= \int dt d^2x\sqrt{g}\,\left[i\hbar\,\psi^\dagger\,D_t\,\psi - \frac{\hbar^2}{2m}g^{ij}\,(D_i\psi)^\dagger(D_j\psi)\right] \\ \exp\,[iS_{eff}] &= \int d\psi\,d\psi^\dagger\exp\,\left[iS[\psi,\psi^\dagger;A_\mu,g_{ij}]\right] \end{aligned}$$

$$S_{eff} = \frac{1}{4\pi}\sum_{s=0}^{N-1}\int\left[[A+(s+\tfrac{1}{2})\omega]\,d[A+(s+\tfrac{1}{2})\omega] - \frac{1}{12}\omega d\omega\right] + \cdots$$

$\omega =$ spin connection $\qquad s = 0 \to LLL\,,\; s = 1 \to$ 1st LL, $\cdots$

$$\frac{\delta S_{eff}}{\delta\omega_0} \sim n_H = \text{Hall viscosity}$$

ABANOV AND GROMOV, 2014

- QHE on S^4 (HU AND ZHANG)

  (background magnetic field = $SU(2)$ "instanton")

Generalization to arbitrary even (spatial) dimensions

- QHE on $\mathbb{CP}^k$ (WITH V.P. NAIR)

$$\mathbb{CP}^k = \frac{SU(k+1)}{U(k)} \qquad (2k \text{ dim space})$$

$U(k) \sim U(1) \times SU(k) \Longrightarrow$ We can have both $U(1)$ and $SU(k)$ background magnetic fields

MATRIX FORMULATION OF LLL DYNAMICS

- QHE on a compact space $M \Longrightarrow$ LLL defines an N-dim Hilbert space

  In the presence of confining potential $\Longrightarrow$ incompressible QH droplet

- Density matrix for ground state droplet : $\hat{\rho}_0$

$$\hat{\rho}_0 = \begin{bmatrix} 1 &&&&&&&& \\ & 1 &&&&&&& \\ && 1 &&&&&& \\ &&& \ddots &&&&& \\ &&&& 1 &&&& \\ &&&&& 0 &&& \\ &&&&&& 0 && \\ &&&&&&& \ddots & \\ &&&&&&&& 0 \end{bmatrix} \begin{array}{l} \updownarrow K \\ \\ \updownarrow N-K \end{array}$$

  K filled states

- Under time evolution: $\hat{\rho}_0 \rightarrow \hat{\rho} = \hat{U}\, \hat{\rho}_0\, \hat{U}^{\dagger}$

  $\hat{U}$ = $N \times N$ unitary matrix ; "collective" variable describing excitations within the LLL

MATRIX FORMULATION OF LLL DYNAMICS (continued)

The action for $\hat{U}$ is

$$S_0 = \int dt \,\mathrm{Tr}\left[i\hat{\rho}_0 \hat{U}^\dagger \partial_t \hat{U} \;-\; \hat{\rho}_0 \hat{U}^\dagger \hat{V} \hat{U} \right]$$

which leads to the evolution equation for density matrix

$$i\frac{d\hat{\rho}}{dt} = [\hat{V}, \hat{\rho}]$$

S_0 has no explicit dependence on properties of space on which QHE is defined, abelian or nonabelian nature of fermions, etc.

S_0 = action of a noncommutative field theory

$$\begin{aligned} S_0 &= \int dt \, \mathrm{Tr} \left[i\hat{\rho}_0 \hat{U}^\dagger \partial_t \hat{U} \; - \; \hat{\rho}_0 \hat{U}^\dagger \hat{V} \hat{U} \right] \\ &= N \int d\mu \, dt \; \left[i(\rho_0 * U^\dagger * \partial_t U) \; - \; (\rho_0 * U^\dagger * V * U) \right] \end{aligned}$$

$$\underbrace{\hat{\rho}_0, \hat{U}, \hat{V}}_{(N\times N)\text{ matrices}} \qquad \Longrightarrow \qquad \underbrace{\rho_0(\vec{x}), U(\vec{x},t), V(\vec{x})}_{\text{symbols}}$$

- $O(\vec{x},t) = \frac{1}{N} \sum_{m,l} \Psi_m(\vec{x}) \hat{O}_{ml}(t) \Psi^*_l(\vec{x})$
- Matrix multiplication $\Longrightarrow$ $*$ product of symbols
- Tr $\Longrightarrow$ $N \int d\mu$

S_0 = exact bosonic action describing the dynamics of LL fermions

SAKITA: 2 dim. context

DAS, DHAR, MANDAL, WADIA,...

RESPONSE TO GAUGE INTERACTIONS

Extend this to include fluctuating gauge fields by gauging S_0

$$\partial_t \to \hat{D}_t = \partial_t + i\hat{\mathcal{A}}$$

$$S = \int dt \, \mathrm{Tr} \left[i\hat{\rho}_0 \hat{U}^\dagger \hat{\partial}_t \hat{U} \; - \; \hat{\rho}_0 \hat{U}^\dagger \hat{V} \hat{U} - \underbrace{\hat{\rho}_0 \, \hat{U}^\dagger \hat{\mathcal{A}} \hat{U}}_{\text{gauge interactions}} \right]$$

In terms of bosonic fields

$$S = N \int dt \, d\mu \, \mathrm{tr} \; \left[i\rho_0 * U^\dagger * \partial_t U \; - \; \rho_0 * U^\dagger * (V + \mathcal{A}) * U \right]$$

QUESTION: How is $\mathcal{A}$ related to the gauge fields coupled to the original fermions?

RESPONSE TO GAUGE INTERACTIONS (continued)

Invariance under $U(N)$ rotations $\delta\hat{U} = -i\hat{\lambda}\,\hat{U}$ implies that S is invariant under

$$\begin{aligned} \delta U &= -i\lambda * U \\ \delta\mathcal{A}(\vec{x},t) &= \partial_t\lambda(\vec{x},t) - i\,(\lambda * (V+\mathcal{A}) - (V+\mathcal{A}) * \lambda) \end{aligned} \tag{1}$$

Since S describes gauge interactions it has to be invariant under usual gauge transformations

$$\delta A_\mu = \partial_\mu\Lambda + i[\,\underbrace{\bar{A}_\mu}_{\text{Background}} + \underbrace{A_\mu}_{\text{Perturbation}}\,,\Lambda]\,, \qquad \delta\bar{A}_\mu = 0 \tag{2}$$

we should choose

$$\begin{aligned} \mathcal{A} &= \text{function}(A_\mu,\ \bar{A}_\mu, V) \\ \lambda &= \text{function}(\Lambda, A_\mu, \bar{A}_\mu) \end{aligned}$$

such that the gauge transformation (2) induces $\delta\mathcal{A}$ in (1) (generalized Seiberg-Witten map)

QHE ON $\mathbb{CP}^k$

Group theoretic analysis:

$$\mathbb{CP}^k = \frac{SU(k+1)}{U(k)} \qquad (2k \text{ dim space})$$

- $U(k) \sim U(1) \times SU(k) \Longrightarrow$ We can have both $U(1)$ and $SU(k)$ background magnetic fields
- Landau wavefunctions are functions on $SU(k+1)$ with particular transformation properties under $U(k)$.
- Each Landau level forms an irreducible $SU(k+1)$ representation, whose degeneracy is easy to calculate.

QHE ON $\mathbb{CP}^k$ (continued)

Wavefunctions are functions on $SU(k+1)$; expressed in terms of Wigner $\mathcal{D}$ functions

$$\Psi \sim \mathcal{D}^{(J)}_{L,R}(g) = \langle\ L\ |\hat{g}|\ R\ \rangle$$

quantum numbers of states in J rep.

Right/left transformations: $\hat{R}_A\,\hat{g} = \hat{g}\,T_A, \qquad \hat{L}_A\,\hat{g} = T_A\,\hat{g}$

- $\hat{R}_a,\ \hat{R}_{k^2+2k} \to$ gauge transformations $\quad (U(k))$
- $\hat{R}_{+i}, \hat{R}_{-i} \to$ covariant derivatives $\quad (i = 1, \cdots, k) \quad [\hat{R}_{+i}, \hat{R}_{-j}] \in U(k)$
- $\hat{L}_A \to$ magnetic translations $\qquad (A \in SU(k+1))$
- How Ψ transforms under $U(k)_R$ depends on choice of background fields
- Choose "constant" $U(1)$ or $U(k)$ background magnetic fields.

$$\bar{F} = d\bar{a} = n\Omega, \quad \Omega = \text{Kahler 2} - \text{form}$$

(Field strengths $\sim U(k)$ structure constants in tangent frame basis $\sim$ Riemannian curvature)

QHE ON $\mathbb{CP}^k$(continued)

$$\Psi^J_{m;\alpha} \sim \langle m \mid \hat{g} \mid \underbrace{R}\ \rangle$$

↘ particular $SU(k)_R$ repr. $\tilde{J}$ with fixed $U(1)_R$ charge $\sim\ n$

$m = 1, \cdots \mathrm{dim} J \Longrightarrow$ counts degeneracy of LL

$\alpha =$ internal gauge index $= 1, \cdots, N' = \mathrm{dim}\tilde{J}$

- Hamiltonian

$$H = \frac{1}{2Mr^2} \sum_{i=1}^{k} \hat{R}_{+i} \hat{R}_{-i} + \text{constant}$$

Lowest Landau level: $\hat{R}_{-i}\Psi = 0$ Holomorphicity condition

($\mid R \rangle$ is lowest weight state)

EDGE EFFECTIVE ACTION

- We know single particle spectrum → calculate symbols, star products
- Calculate S_0 for large N, K with $N \gg K \gg 1$ ($n \to \infty$ limit)

$$S_0 = N \int d\mu \, dt \, \left[i(\rho_0 * U^\dagger * \partial_t U) \; - \; (\rho_0 * U^\dagger * V * U) \right]$$

A. Abelian background magnetic field $U(1)$

- symbol of $\hat{X}$ → function X
- $X * Y = XY + \frac{1}{n}\left(g^{ij} + \frac{i}{2}(\Omega^{-1})^{ij}\right)\partial_i X \partial_j Y + \cdots$
- $\left([\hat{X}, \; \hat{Y}]\right)_{symbol} \to \frac{i}{n}\{X(\vec{x}, t), \; Y(\vec{x}, t)\}_{\text{PB}} = \frac{i}{n}(\Omega^{-1})^{ij} \, \partial_i X(\vec{x}, t) \, \partial_j Y(\vec{x}, t)$
- $\rho_0 \to \Theta\left(R_D^2 - r^2\right)$, $\qquad R_D$= droplet radius

EDGE EFFECTIVE ACTION (continued)

$$S_0 \sim \int_{\partial D} (\partial_t \phi + u\, \mathcal{L}\phi)\mathcal{L}\phi$$

This is a $(2k-1)$ (space) dimensional chiral action defined on the droplet boundary, with

$$\mathcal{L}\phi = (\Omega^{-1})^{ij}\hat{r}_j \partial_i \phi, \qquad \mathcal{L} = \begin{cases} \text{derivative along boundary of droplet} \\ \quad \to \partial_\theta \text{ in 2 dim.} \end{cases}$$

B. Nonabelian background magnetic field $U(k)$

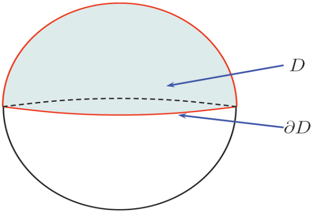
- Symbol of a matrix = $(N' \times N')$ matrix valued function
  $\hat{X} \Longrightarrow X_{\alpha\beta}(\vec{x},t) = \frac{1}{N}\sum_{m,l} \Psi_{m;\alpha}(\vec{x}) X_{ml}(t) \Psi^*_{l;\beta}(\vec{x})$
  $\alpha, \beta = 1, \cdots N'$

EDGE EFFECTIVE ACTION (continued)

Bosonic action can be written in terms of $G \in U(N')$

$$\begin{aligned} S_0 =& \frac{1}{4\pi}\int_{\partial D} \mathrm{tr}\left[\left(G^\dagger \dot{G} + u\, G^\dagger \mathcal{L} G\right) G^\dagger \mathcal{L} G\right] \\ &+ \frac{1}{4\pi}\int_D \mathrm{tr}\Big[-d\left(i\bar{A} dG G^\dagger + i\bar{A} G^\dagger dG\right) + \underbrace{\frac{1}{3}\left(G^\dagger dG\right)^3\Big]\left(\frac{\Omega}{2\pi}\right)^{k-1}\frac{1}{(k-1)!}}_{\text{WZW-term in } 2k+1 \text{ dim}} \\ \equiv& S_{\mathrm{WZW}}(A^L = A^R = \bar{A}) \end{aligned}$$

$\mathcal{L} = (\Omega^{-1})^{ij}\hat{r}_j D_i$ = covariant derivative along the boundary of droplet



BOSONIC ACTION WITH GAUGE INTERACTIONS

- In the presence of gauge fluctuations (DK)

$$\begin{aligned} S &= N\int dt\, d\mu\, \mathrm{tr}\, \left[i\rho_0 * U^\dagger * \partial_t U \;-\; \rho_0 * U^\dagger * (V + \mathcal{A}) * U\right] \\ &= S_{\text{edge}} + S_{\text{bulk}} \end{aligned}$$

$S_{\text{edge}} \sim S_{\text{WZW}}\left(A^L = A + \bar{A}\,, A^R = \bar{A}\right)$ = Chirally gauged WZW action in $2k$ dim

$S_{\text{bulk}} \sim S_{CS}^{2k+1}(\tilde{A}) + \cdots$ = $(2k+1)$ dim CS action

Here $\tilde{A} = (A_0 + V, \bar{a}_i + \bar{A}_i + A_i)$ = background + fluctuations

- Gauge Invariance $\Longrightarrow$ Anomaly Cancellation

$$\delta S_{\text{edge}} \neq 0, \qquad \delta S_{\text{bulk}} \neq 0$$

$$\delta S_{\text{edge}} + \delta S_{\text{bulk}} = 0$$

WHAT ABOUT METRIC PERTURBATIONS?

- Universal matrix action describing dynamics within each Landau level
- single-particle spectrum + large N limit $\Longrightarrow$

  (bulk + edge) effective actions
- gauge invariance is automatically built in
- Questions:
  - How important is precise knowledge of single particle spectrum?
  - Can we deviate from $\mathbb{CP}^k$?
  - How do we introduce metric perturbations?

- The lowest Landau level obeys the holomorphicity condition

$$\hat{R}_{-i}\Psi = 0$$

- The number of normalizable solutions is given by the Dolbeault index.
- The Dolbeault index is given by

$$\mathrm{Index}(\bar{\partial}_V) = \int_M \mathrm{td}(T_C M) \wedge \mathrm{ch}(V)$$

$\mathrm{td}(T_c M) =$ Todd class (for complex tangent space) = given in terms of traces of curvatures

ch (V) = Chern character = Tr $\left(e^{iF/2\pi}\right)$

- For a fully filled LLL (each particle carries $e = 1$):

  degeneracy = charge $\implies$ index density = charge density

- So we can use

$$\frac{\delta S_{\mathit{eff}}}{\delta A_0} = J_0 = \text{Dolbeault Index Density}$$

and integrate up to get S_{eff}.

- In 2+1 dimensions
$$\mathrm{Index}(\bar\partial_V) = \int_M \frac{iF}{2\pi} + \frac{iR}{4\pi}$$
- Consider QHE on $\mathbb{CP}^1 = SU(2)/U(1) \sim S^2$

  The background values for the curvatures are
$$\bar F = -in\Omega, \quad \bar R|_{T_M K} = -i2\Omega, \quad \Omega = i\left[\frac{dzd\bar z}{1+z\bar z} - \frac{\bar z dz\, z\cdot d\bar z}{(1+z\bar z)^2}\right]$$
  degeneracy of LLL= $\mathrm{Index}(\bar\partial_V) = n+1$ as expected HALDANE
- Charge density including fluctuations
$$J_0 = \frac{iF}{2\pi} + \frac{iR}{4\pi}$$
- This leads to the effective action
$$S^{LLL}_{3d} = \frac{i^2}{4\pi}\int A\Big[F+R\Big] + S_{\text{grav}} + \cdots$$

- For higher Landau levels there is no holomorphicity condition, Dolbeault index is problematic
- The wave functions in the s-th LL for $\mathbb{CP}^1 = SU(2)/U(1) \sim S^2$ are

  $$\Psi_m(g) \sim \langle J, m|g|J, -n/2\rangle, \qquad J = n/2 + s$$

  They satisfy $R_3\Psi = -n/2\ \Psi$, but do not satisfy holomorphicity $R_-\Psi \neq 0$
- The lowest weight state in the same representation

  $$\tilde{\Psi}_m(g) \sim \langle J, m|g|J, -n/2 - s\rangle$$

  which satisfy the holomorphicity condition $R_-\tilde{\Psi} = 0 \Longrightarrow$ LLL

- It couples to a $U(1)$ background field

$$\bar{\mathcal{F}} = -i\big(n + 2s\big)\Omega = \bar{F} + s\bar{R} = \bar{F} + \bar{\mathcal{R}}_s$$

- So we have a mapping

  Particle with spin-0 in s-th LL $\Longrightarrow$ Particle with spin-s in LLL
- counting of states same $\Longrightarrow$ use Dolbeault index for degeneracy
- Chern character : $\mathrm{Tr}\left(e^{iF/2\pi}\right) \to \mathrm{Tr}\left(e^{i(\mathcal{R}_s+F)/2\pi}\right)$

- For s-th Landau level on $\mathbb{CP}^1 = SU(2)/U(1) \sim S^2$ we find

$$\text{Index}(\bar{\partial}_V) = \int \left[\frac{iF}{2\pi} + (s + \tfrac{1}{2}) \frac{iR}{2\pi} \right]$$

- S_{eff} can be determined from this

$$S_{3d}^{(s)} = \frac{i^2}{4\pi} \int A \left[F + (2s+1)R \right] + S_{\text{grav}} + \cdots$$

- The purely gravitational term can be obtained from the gravitational part of the index density in $2k+2$ dim following the descent approach

$$[\text{Index Density}]_{2k+2} = d\left[\cdots \right]$$

- Derive the full topological effective action

$$S_{3d}^{(s)} = \frac{i^2}{4\pi} \int \left\{ \Big[A + (s + \tfrac{1}{2})\,\omega \Big] d \Big[A + (s + \tfrac{1}{2})\,\omega \Big] - \tfrac{1}{12} \omega \, d\omega \right\}$$

This agrees with ABANOV,GROMOV; KLETSOV, MA, MARINESCU, WIEGMANN; BRADLYN, READ; CAN, LASKIN, WIEGMANN

HIGHER DIMENSIONS

- In $2k+1$ dimensions (fluctuations around $\mathbb{CP}^k = SU(k+1)/U(k)$ background fields)
  - We have curvatures in the algebra of $U(k)$

$$R = -i\left[R^0\mathbf{1} + \tilde{R}^a t_a\right], \qquad R^0 = d\omega^0, \qquad \tilde{R} = d\tilde{\omega} + \tilde{\omega} \wedge \tilde{\omega}$$

  - Abelian and/or nonabelian gauge fields
  - Nonzero spin to include higher Landau levels

$$\mathcal{R}_s = -i\left[sR^0\mathbf{1} + \tilde{R}^a T_a\right]$$

  - The index theorem is now

$$\mathrm{Index}(\bar{\partial}_V) = \int_M \mathrm{td}(T_c M) \wedge \mathrm{Tr}\left(e^{i(\mathcal{R}_s + F)/2\pi}\right)$$

FULL TOPOLOGICAL EFFECTIVE ACTION

The full topological effective action is

$$S^{(s)}_{2k+1} = \int \left[\mathrm{td}(T_c K) \wedge \sum_p (CS)_{2p+1}(\omega_s + A) \right]_{2k+1} + 2\pi \int \Omega^{\mathrm{grav}}_{2k+1}$$

where

$$\left[\mathrm{td}(T_c K) \wedge \mathrm{ch}(S)\right]_{2k+2} = d\,\Omega^{\mathrm{grav}}_{2k+1} + \frac{1}{2\pi}\, d \left[\mathrm{td}(T_c K) \wedge \sum_p (CS)_{2p+1}(\omega_s) \right]_{2k+1}$$

Gives correct expressions for degeneracies in all cases we know QHE wavefunctions: $\mathbb{CP}^k$ (abelian and nonabelian gauge fields), $S^2 \times S^2$, etc

EXAMPLES

- 2+1 dim, s-th Landau level

$$S_{3d}^{(s)} = \frac{i^2}{4\pi} \int \left\{ \left[A + (s + \frac{1}{2})\, \omega \right] d \left[A + (s + \frac{1}{2})\, \omega \right] - \frac{1}{12} \omega \, d\omega \right\}$$

- 4+1 dim, s-th Landau level

  $\mathbb{CP}^2 = SU(3)/U(2)$; $U(1)$ gauge field; $U(2)$ spin connection and curvature

$$\begin{aligned} S_{5d}^{(s)} \quad = \quad & \frac{i^3 (s+1)}{(2\pi)^2} \int \left\{ \frac{1}{3!} \Big(A + (s+1)\omega^0 \Big) \Big[d \Big(A + (s+1)\omega^0 \Big) \Big]^2 \right. \\ & \left. - \frac{1}{12} \Big(A + (s+1)\omega^0 \Big) \left[(d\omega^0)^2 - \left[((s+1)^2 - \frac{3}{2} \right] \mathrm{Tr}(\tilde{R} \wedge \tilde{R}) \right] \right\} \end{aligned}$$

- 6+1 dim, lowest Landau level

  $\mathbb{CP}^3 = SU(4)/U(3)$; $U(1)$ gauge field; $U(3)$ curvature

$$\begin{aligned}
S_{7d}^{\mathrm{LLL}} &= \frac{1}{(2\pi)^3}\int\Bigg\{\frac{1}{4!}\left(A+\frac{3}{2}\omega^0\right)\left[d\left(A+\frac{3}{2}\omega^0\right)\right]^3 \\
&\quad -\frac{1}{16}\left(A+\frac{3}{2}\omega^0\right)d\left(A+\frac{3}{2}\omega^0\right)\left[(d\omega^0)^2+\frac{1}{3}\mathrm{Tr}(\tilde{R}\wedge\tilde{R})\right] \\
&\quad +\frac{1}{1920}\omega^0 d\omega^0\left[17(d\omega^0)^2+14\mathrm{Tr}(\tilde{R}\wedge\tilde{R})\right]+\frac{1}{720}\omega^0\mathrm{Tr}(\tilde{R}\wedge\tilde{R}\wedge\tilde{R})\Bigg\} \\
&\quad +\frac{1}{120}\int(CS)_7(\tilde{\omega})
\end{aligned}$$

SUMMARY

- Extend QHE to higher dimensions; physical realizations of fuzzy spaces
- Universal matrix action → noncommutative field theory description of LL dynamics
- At large N limit action describes dynamics of abelian/nonabelian quantum Hall droplets with gauge fluctuations
- anomaly free bulk/edge dynamics
- Use index theorems to introduce metric perturbations
- Applications to fluids and higher dim transport coefficients