Exactness from symmetries in interacting current algebra theories

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Based on work with:

- G. Georgiou and K. Siampos:
  arXiv:1509.02946, arXiv:1604.08212 & arXiv:1610.05314
- G. Georgiou: arXiv:1612.05012
- G. Georgiou, E. Sagkrioti and K. Siampos: arXiv:1703.00462

Motivation

- Exact beta-function and anomalous dimensions.
  - The quantum behaviour of Physical systems is encoded in the beta-function of the anomalous dimensions of operators.
  - Traditionally these are computed perturbatively. It is a rare occasion to be able to compute them exactly.
- Systematic construction of new (integrable) deformations of CFT's representing interacting quantum field theories.
- In conjunction with a corresponding gravitational approach use them in an AdS/CFT context.

Outline

- Define the type of theories of interest
- Explain the symmetries to be used.
- Technical points of the computations.
- Some perturbative info plus symmetry and analyticity considerations lead to exact results.
- Concluding remarks

The theories of interest: Self-interacting theories

Consider any 2-dim CFT with action S_k and a group G structure having holomorphic & anti-holomorphic currents $J^a(z)$ & $\bar{J}^a(\bar{z})$, obeying

$$J^a(z)J^b(w) = \frac{\delta_{ab}}{(z-w)^2} + \frac{f_{abc}}{\sqrt{k}}\frac{J^c(w)}{z-w} + \cdots$$

and similarly for the $\bar{J}^a(\bar{z})$'s.

- We are interested in studying the theory away from the conformal point driven by a self-interaction current bilinear

$$S_{k,\lambda} = S_k - \frac{1}{\pi}\int d^2z\ \lambda_{ab}J^a\bar{J}^b\ .$$

- This is of interest in string theory, AdS/CFT correspondence, supergravity and fermionic systems.

In particular:

- We are interested in computing the RG flow eqs

$$\mu \frac{d\lambda_{ab}}{d\mu} = \cdots \ .$$

- The currents' anomalous dims, as functions of λ_{ab} and k, as well as the anomalous dims of all operators.
- Search for new fixed points under the RG flow towards the IR.
- We would like to do that exactly in λ and in k.
- Unlike traditional approaches these computations non-perturbative.

Effective action and symmetry

- A certain procedure gives the σ-model action [KS 13]

$$S_{k,\lambda}(g) = S_{\mathrm{WZW},k}(g) + \frac{k}{\pi}\int d^2\sigma\ J_+^a(\lambda^{-1} - D^T)^{-1}_{ab} J_-^b\ ,$$

where $g \in G$ and

$$J_+^a = -i\mathrm{Tr}(t^a \partial_+ g g^{-1})\ , \qquad J_-^a = -i\mathrm{Tr}(t^a g^{-1}\partial_- g)\ ,$$
$$D_{ab} = \mathrm{Tr}(t_a g t_b g^{-1})\ .$$

- This is the effective action for current-current interactions of the same CFT. For small λ_{ab}

$$S_{k,\lambda}(g) = S_{\mathrm{WZW},k}(g) + \frac{k}{\pi}\int d^2\sigma\ \lambda_{ab} J_+^a J_-^b + \dots\ .$$

- This theory has a duality-type symmetry [Itsios-KS-Siampos 14]

$$k \to -k\ , \qquad \lambda \to \lambda^{-1}\ , \qquad g \to g^{-1}\ .$$

- It should be reflected as a symmetry in physical quantities.

β-function and anomalous dimensions [Georgiou-Siampos-KS 15 & 16]

Perturbative computations

The simplest case corresponding to an integrable theory [KS 13, Hollowood-Miramontes-Schmidtt 14]

$$\lambda_{ab} = \lambda \delta_{ab} \ .$$

- We want to compute the 2-point functions

$$\langle J^a(x_1) J^b(x_2) \rangle_\lambda = \langle J^a(x_1) J^b(x_2) e^{-\frac{\lambda}{\pi}\int d^2 z J^a(z) \bar{J}^a(\bar{z})} \rangle \ ,$$
$$\langle J^a(x_1) \bar{J}^b(x_2) \rangle_\lambda = \langle J^a(x_1) \bar{J}^b(x_2) e^{-\frac{\lambda}{\pi}\int d^2 z J^a(z) \bar{J}^a(\bar{z})} \rangle \ ,$$

  perturbatively in λ by expanding the exponential.
- The basic correlators are

$$\langle J^a(x_1) J^b(x_2) \rangle = \frac{\delta_{ab}}{x_{12}^2} \ , \quad \langle J^a(x_1) J^b(x_2) J^c(x_3) \rangle = \frac{1}{\sqrt{k}} \frac{f_{abc}}{x_{12} x_{13} x_{23}}$$

  and similarly for the $\bar{J}^a$'s. Mixed $J\bar{J}$ correlators vanish.
- For higher correlators use Ward dentities

Perturbative results; Renormalization

Relations between the bare and renormalized quantities

$$J_0^a = Z^{1/2} J^a \ , \qquad \bar{J}_0^a = Z^{1/2} \bar{J}^a \ , \qquad \lambda_0 = Z_1 \lambda$$

- The renormalized n-point functions are cutoff independent

$$\langle J^a(x_1) J^b(x_2) \rangle_\lambda = Z^{-1} \langle J_0^a(x_1) J_0^b(x_2) \rangle_{Z_1 \lambda} \ .$$

- Up to three-loops this requires that

$$Z^{-1} = 1 + 2c_G \lambda^3 - \frac{c_G}{k} \left(\lambda^2 - 2\lambda^3 + \mathcal{O}(\lambda^4) \right) \ln(\varepsilon^2 \mu^2) \ ,$$

$$Z_1 = 1 + \frac{c_G}{k} \left(\frac{1}{2}\lambda - \lambda^2 + \mathcal{O}(\lambda^3) \right) \ln(\varepsilon^2 \mu^2) \ .$$

Depends on the energy scale μ and a small distance cut-off.

The perturbative β-function and anomalous dimensions

- The β-function is by definition

$$\beta = \frac{1}{2}\mu\frac{d\lambda}{d\mu} = -\frac{c_G}{2k}\left(\lambda^2 - 2\lambda^3 + \mathcal{O}(\lambda^4)\right) \ ,$$

where the bare coupling λ_0 is kept fixed.

- The anomalous dimension of the currents is

$$\gamma^{(J)} = \mu\frac{d\ln Z^{1/2}}{d\mu} = \frac{c_G}{k}\left(\lambda^2 - 2\lambda^3 + \mathcal{O}(\lambda^4)\right) \ .$$

Task: Extend these exactly in λ?

Analyticity: λ-dependence of physical quantities

- Expand the action for $g = e^{ix^a t^a}$ around the identity

$$S_{k,\lambda} = \frac{k}{4\pi}\frac{1+\lambda}{1-\lambda}\int \partial_+ x^a \partial_- x^a + \cdots$$

- The β-function & anomalous dims may have poles at $\lambda = \pm 1$.
- The β-function & anomalous dims should be invariant under

$$k \to -k \ , \qquad \lambda \to \frac{1}{\lambda} \ ,$$

  for $k \gg 1$.
- Perturbative information to $\mathcal{O}(\lambda^2)$ and the above symmetry are enough to determine the β-function and the anomalous dimensions exactly in λ and to leading order in k.

The exact β-function and anomalous dimensions

The exact β-function and anomalous dimensions are of the form

$$\beta_\lambda = -\frac{c_G}{2k}\frac{f(\lambda)}{(1+\lambda)^2}\ , \qquad \gamma^{(J)} = \frac{c_G}{k}\frac{g(\lambda)}{(1-\lambda)(1+\lambda)^3}\ ,$$

where $f(\lambda)$ and $g(\lambda)$ are analytic in λ.

- They have a well defined non-Abelian and pseudodual limits.
- Due to the symmetry $(k,\lambda)\mapsto(-k,\lambda^{-1})$ we have that

  $$\lambda^4 f(1/\lambda) = f(\lambda)\ , \qquad \lambda^4 g(1/\lambda) = g(\lambda)\ ,$$

  where $f(\lambda)$ and $f(\lambda)$ are polynomials of, at most, degree four of the form

  $$f(\lambda) = a_0(1+\lambda^4) + a_2\lambda^2 + a_1(\lambda+\lambda^3)$$

  and similarly for $g(\lambda)$.
- The coefficients can be fixed by the above symmetry and by matching with the two-loop perturbative result.

- The final result is

$$\beta_\lambda = -\frac{c_G}{2k}\frac{\lambda^2}{(1+\lambda)^2} \leqslant 0$$

  and

$$\gamma^{(J)} = \frac{c_G}{k}\frac{\lambda^2}{(1-\lambda)(1+\lambda)^3} \geqslant 0\ .$$

  Agree with perturbation theory to order checked, i.e. $\mathcal{O}(\lambda^3)$ and $\mathcal{O}(\lambda^4)$.
- Results also agree with gravity computations [Itsios-KS-Siampos 14, KS-Siampos 15] (see Siampos' talk).
- For other fields analogous computations have been performed.

3-point functions of currents

With similar computations and arguments we compute:

$$\langle J^a(x_1)J^b(x_2)J^c(x_3)\rangle = \frac{1+\lambda+\lambda^2}{\sqrt{k(1-\lambda)(1+\lambda)^3}}\frac{f_{abc}}{x_{12}x_{13}x_{23}} .$$

and

$$\langle J^a(x_1)J^b(x_2)\bar{J}^c(\bar{x}_3)\rangle = \frac{\lambda}{\sqrt{k(1-\lambda)(1+\lambda)^3}}\frac{f_{abc}\bar{x}_{12}}{x_{12}^2\bar{x}_{13}\bar{x}_{23}} .$$

- These are leading order for $k \gg 1$ and respect the symmetry

$$k \to -k , \qquad \lambda \to \frac{1}{\lambda} .$$

- The other correlators follow from parity.
  Similarly one computes correlators involving primary fields.

Left-right asymmetric deformations

So far no new fixed point towards the IR.
This changes when two different levels k_L and k_R.

- Beta-function

$$\frac{d\lambda}{dt} = -\frac{c_G}{2\sqrt{k_L k_R}} \frac{\lambda^2(\lambda - \lambda_0)(\lambda - \lambda_0^{-1})}{(1-\lambda^2)^2} \ .$$

A new fixed point in the IR at $\lambda = \lambda_0 = \sqrt{\frac{k_L}{k_R}}$.

- Anomalous dimensions

$$\gamma_L = \frac{c_G}{k_R} \frac{\lambda^2(\lambda - \lambda_0^{-1})^2}{(1-\lambda^2)^3} \ , \quad \bar{\gamma}_R = \frac{c_G}{k_L} \frac{\lambda^2(\lambda - \lambda_0)^2}{(1-\lambda^2)^3} \ .$$

- Evidence of the RG flow to a diffenent CFT in the IR

$$G_{k_L} \times G_{k_R} \quad \overset{\text{IR}}{\Longrightarrow} \quad \frac{G_{k_L} \times G_{k_R - k_L}}{G_{k_R}} \times G_{k_R - k_L} \ .$$

For $G = SU(2)$ argued to describe a fermi liquid as the IR fixed point of interacting chiral fermions [Andrei-Douglas-Jerez 99]

The theories of interest: Interacting theories

We are interested in the theory resulting from current-current interactions of different CFTs

$$S_{k,\lambda}(g_1, G_2) = S_k(g_1) + S_k(g_2) + \frac{k}{\pi} \int \lambda_{1,ab} J^a_{1+} J^b_{2-} + \lambda_{2,ab} J^a_{2+} J^b_{1-} + \cdots \, .$$

- The theory is driven away from the conformal point.

$$\mu \frac{d\lambda_i}{d\mu} = \cdots \, , \quad i = 1, 2 \, .$$

- Using an effective action [Georgiou-KS 16] one finds the duality-type symmetry

$$k \to -k, \quad \lambda_1 \to \lambda_1^{-1} \, , \quad \lambda_2 \to \lambda_2^{-1}, \quad g_1 \leftrightarrow g_2^{-1} \, .$$

- For

$$\lambda_{i,ab} = \lambda_i \delta_{ab} \, , \quad i = 1, 2 \, ,$$

  the theory is integrable [Georgiou-KS, 16].

β-function and anomalous dimensions

Some surprising results:

- The current correlators can be obtained from two copies each of which corresponds to a self-interacting λ-deformed model, one with coupling λ_1 and the other with λ_2.
- Hence the β-function and the current anomalous dimensions are the same as if we had two independent self-interacting λ-models.
- The above is only true when restricted to correlation functions involving currents exclusively; not for generic primary fields.
- The above assertion is valid for all values of the deformation matrices λ_1 and λ_2 and to all order in level k.

There is a deep underlying relation between these models via canonical transformations in phase space [Georgiou-KS-Siampos, to appear].

Concluding remarks

- Computed exactly the beta-function and anomalous dimensions of operators in interacting current algebra theories.
- Computations are based on leading order perturbative results and symmetry arguments.
- New integrable theories, as current-current interactions of two exact CFT WZW models.
- There are canonical equivalences among these models, exaplainiíã identical expressions for the beta-functions and current's anomalous dimensions.
- Future direction 1: Embed to type-II supergravity as in [KS-Thompson 14, Demulder-KS-Thompson 15, Borsato-Tseytlin 15]. Use them in an AdS/CFT context.
- Future direction 2: New non-Abelian type T-dualities: The basic idea [Georgiou-KS-Siampos, in progress] is to expand the product $g_1 g_2$ around the identity and take the $k \to \infty$ limit.
- Future direction 3: Consider chains and/or webs of models (cyclic or infinite) [in progress].