Parton Distribution Functions for TensorGluons in Generalised Yang-Mills Theory

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Beyond the Standard Model ——————————————-
Extension of Poincaré Group and of Yang-Mills Theory

Proton Structure and PDF for Tensorgluons.

SUSY and Yangian symmetry of the Splitting Probabilities

1.Generalization of Yang-Mills theory
Phys. Lett. B 625 (2005) 341

2.Extension of Poincaré Group and Tensor Gauge Fields
Int.J.Mod.Phys. A25 (2010) 5765-5785

3.Supersymmetric Extensions of the Poincare group
Ignatios Antoniadis, Lars Brink, G.S.
J.Math.Phys. 52 (2011) 072303,

4.Asymptotic Freedom of non-Abelian Tensor Gauge Fields
Phys. Lett. B 732 (2014) 150

5.Yangian and SUSY symmetry of High Spin Parton Amplitudes
Roland Kirschner and G.S.
ArXiv:1701.06660 [hep-th]

The extension of the Poincaré algebra

$$
\begin{aligned}
&[P^{\mu},\ P^{\nu}] = 0,\\
&[M^{\mu\nu},\ P^{\lambda}] = i(\eta^{\lambda\nu}\ P^{\mu} - \eta^{\lambda\mu}\ P^{\nu}),\\
&[M^{\mu\nu},\ M^{\lambda\rho}] = i(\eta^{\mu\rho}\ M^{\nu\lambda} - \eta^{\mu\lambda}\ M^{\nu\rho} + \eta^{\nu\lambda}\ M^{\mu\rho} - \eta^{\nu\rho}\ M^{\mu\lambda}),
\end{aligned}
$$

$$
\begin{aligned}
&[P^{\mu},\ L_a^{\lambda_1\ldots\lambda_s}] = 0,\\
&[M^{\mu\nu},\ L_a^{\lambda_1\ldots\lambda_s}] = i(\eta^{\lambda_1\nu} L_a^{\mu\lambda_2\ldots\lambda_s} + \ldots + -\eta^{\lambda_s\mu} L_a^{\lambda_1\ldots\lambda_{s-1}\nu}),
\end{aligned}
$$

$$
[L_a^{\lambda_1\ldots\lambda_n}, L_b^{\lambda_{n+1}\ldots\lambda_s}] = i f_{abc} L_c^{\lambda_1\ldots\lambda_s} \quad (s = 0, 1, 2, \ldots).
$$

The generators $L_a^{\lambda_1\ldots\lambda_s}$ carry *higher spins and charges*.

The non-Abelian tensor gauge fields are defined by the relation:

$$\mathcal{A}_{\mu}(x, L) = \sum_{s=0}^{\infty} \frac{1}{s!} \; A^{a}_{\mu\lambda_1 \ldots \lambda_s}(x) \; L_a^{\lambda_1 \ldots \lambda_s}. \tag{1}$$

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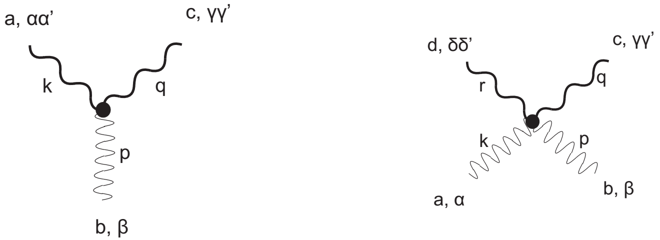
The new Lagrangian of extended Yang-Mills theory is

$$\begin{aligned}\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + = & - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} \\ & - \frac{1}{4} G^a_{\mu\nu,\lambda} G^a_{\mu\nu,\lambda} - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu,\lambda\lambda} + \\ & + \frac{1}{4} G^a_{\mu\nu,\lambda} G^a_{\mu\lambda,\nu} + \frac{1}{4} G^a_{\mu\nu,\nu} G^a_{\mu\lambda,\lambda} + \frac{1}{2} G^a_{\mu\nu} G^a_{\mu\lambda,\nu\lambda} + \end{aligned}$$

Helicity spectrum of the tensorgluons

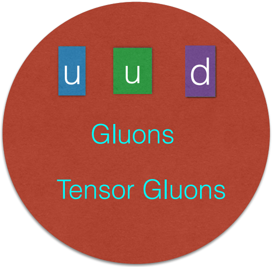
$$
\begin{array}{llllll}
\pm 1 & & & & & \\
\pm 2, & 0 & & & & \\
\pm 3, & \pm 1, & \pm 1 & & & \\
\pm 4, & \pm 2, & \pm 2, & 0 & & \\
\pm 5, & \pm 3, & \pm 3, & \pm 1, & \pm 1 & \\
\pm 6, & \pm 4, & \pm 4, & \pm 2, & \pm 2, & 0 \\
\multicolumn{6}{l}{\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots,}
\end{array}
$$



Interaction Vertices of gluons and tensorgluons are with dimensionless coupling constant g

What we need to know - are the Parton Distribution Functions - PDF

for the new partons $\rightarrow$ TensorGluons



Radiation of tensorgluons by gluons, the new evolution equations

$$\dot{q}^i_t(x) = \frac{\alpha(t)}{2\pi} \int_x^1 \frac{dy}{y} [q^j(y,t)\ P_{q^i q^j} + G(y,t)\ P_{q^i G}], \qquad (2)$$

$$\dot{G}_t(x) = \frac{\alpha(t)}{2\pi} \int_x^1 \frac{dy}{y} [q^j(y,t) P_{G q^j} + G(y,t) P_{GG} + T_r(y,t) P_{G_r T}],$$

$$\dot{T}_r(x) = \frac{\alpha(t)}{2\pi} \int_x^1 \frac{dy}{y} [G(y,t)\ P_{T_r G} + \sum_s T_s(y,t)\ P_{T_r T_s}].$$

The $\alpha(t)$ is the running coupling constant ($\alpha = g^2/4\pi$)

$$\alpha(t) = \frac{\alpha}{1 + b_1\ \alpha\ t}, \qquad (3)$$

where b_1 is the one-loop Callan-Symanzik beta coefficient,

$$b_1 = b_{quarks} + b_{gluons} + b_{tensorgluons}$$

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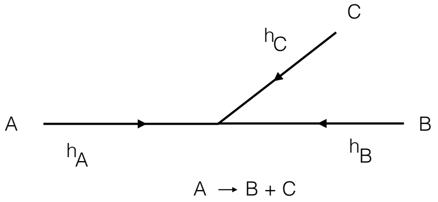
$P^C_{BA}(z)$ describes the probability of finding a particle B inside a particle A with fraction z of the longitudinal momentum of A and radiation of the third particle C with fraction $(1-z)$ of the longitudinal momentum of A

$$P^C_{BA}(z) = \frac{1}{2} z(1-z) \bar{\sum}_{helicities} \frac{|M_{A\to B+C}|^2}{p^2_\perp}, \qquad (4)$$

where a sum is over the helicities of B and C and an average over the helicity of A.



The symmetry properties of the $P^{C}_{BA}(z)$ over exchange $B \leftrightarrow C$

$$P^{C}_{BA}(z) = P^{B}_{CA}(1-z) \tag{5}$$

and a crossing relation

$$P^{C}_{AB}(z) = (-1)^{2h_A+2h_B+1} z P^{C}_{BA}(\frac{1}{z}), \tag{6}$$

which emerges because of the time reversal $A \leftrightarrow B$.

The DGLAP quark and gluon Splitting Probabilities in QCD are:

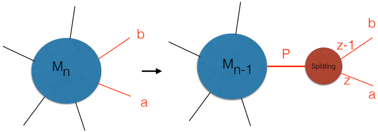
$$\begin{aligned} P_{qq}(z) &= C_2(R)\frac{1+z^2}{1-z}, \\ P_{Gq}(z) &= C_2(R)\frac{1+(1-z)^2}{z}, \\ P_{qG}(z) &= T(R)[z^2+(1-z)^2], \\ P_{GG}(z) &= C_2(G)\left[\frac{1}{z(1-z)}+\frac{z^4}{z(1-z)}+\frac{(1-z)^4}{z(1-z)}\right], \end{aligned} \tag{7}$$

where $C_2(G) = N, C_2(R) = \frac{N^2-1}{2N}, T(R) = \frac{1}{2}$ for the SU(N).

The splitting probabilities of **gluons to tensorgluons** is

$$
\begin{aligned}
P_{T_sG}(z) &= C_2(G)\left[\frac{z^{2s+1}}{(1-z)^{2s-1}}+\frac{(1-z)^{2s+1}}{z^{2s-1}}\right],\\
P_{GT_s}(z) &= C_2(G)\left[\frac{1}{z(1-z)^{2s-1}}+\frac{(1-z)^{2s+1}}{z}\right],\\
P_{T_sT_s}(z) &= C_2(G)\left[\frac{z^{2s+1}}{(1-z)}+\frac{1}{(1-z)z^{2s-1}}\right].
\end{aligned}
$$

where s is the spin of the tensorgluons. I.Antoniadis, G.S. (2012)



To find **tensor-tensor splitting probabilities** one should use spinor representation of the interaction vertex M_3

$$P(z) = \frac{1}{2} z(1-z)|M_3|^2 \frac{1}{|w|^2}$$

$$M_3^- = g f^{abc} < 1,2 >^{-2h_1-2h_2-1} < 2,3 >^{2h_1+1} < 3,1 >^{2h_2+1},$$

.

$$M_3^+ = g f^{abc} [1,2]^{2h_1+2h_2-1} [2,3]^{-2h_1+1} [3,1]^{-2h_2+1},$$

f^{abc} are the structure constants.

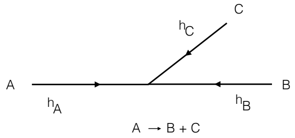


Figure : Splitting of the tensorgluons P^{C}_{BA}.

$$P^{h_C}_{h_B h_A} = \frac{1}{z^{2\eta h_B - 1}(1-z)^{2\eta h_C - 1}}, \quad h_C + h_B + h_A = \eta = \pm 1.$$

The formula describes all known splitting probabilities found earlier in QFT and the generalised Yang-Mills theory.

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$$P^{h_C}_{h_B h_A} = \frac{1}{z^{2\eta h_B - 1}(1-z)^{2\eta h_C - 1}}, \quad h_C + h_B + h_A = \eta = \pm 1.$$

let us consider the examples:

$h_A = +1/2,\ h_B = -1/2,\ h_C = +1$ then $h_C + h_B + h_A = \eta = +1$
$h_A = -1/2,\ h_B = +1/2,\ h_C = +1$ then $h_C + h_B + h_A = \eta = +1$

$$P^{+1}_{-1/2+1/2} = \frac{1}{z^{2(-1/2)-1}(1-z)^{2(+1)-1}} = \frac{z^2}{(1-z)}$$

$$P^{+1}_{+1/2-1/2} = \frac{1}{z^{2(+1/2)-1}(1-z)^{2(+1)-1}} = \frac{1}{(1-z)}.$$

so that unpolarised splitting probability will be

$$P_{qq}(z) = C_2(R)\frac{1+z^2}{1-z}.$$

The $\mathcal{N}=1$ Kounnas-Ross relations in SUSY QCD are

$$\begin{aligned} P_{GG} + P_{\lambda G} &= P_{G\lambda} + P_{\lambda\lambda} \\ P_{Gq} + P_{\lambda q} &= P_{Gs} + P_{\lambda s} \\ P_{qG} + P_{sG} &= P_{q\lambda} + P_{s\lambda} \\ P_{qq} + P_{sq} &= P_{qs} + P_{ss}. \end{aligned}$$

It is interesting to check if the high spin evolution kernels $P^{h_C}_{h_B h_A}$ fulfil generalised $\mathcal{N}=1$ supersymmetry relations.

The supermultiplets $(1, 1/2)$ and $(s, s-1/2)$ of the gluons-gluinos and tensorgluons-tensorgluionos fulfil

$$P^{1/2}_{s\ s-1/2} + P^{1}_{s-1/2\ s-1/2} = P^{1/2}_{s-1/2\ s} + P^{1}_{ss}.$$

Consider two arbitrary supermultiplets
$(s, s-1/2)$ and $(r, r-1/2)$.

For these supermultiplets the $\mathcal{N}=1$ SUSY relation has the following generalised Kounnas-Ross form

$$P_{r\ s-1/2}^{s-r+1/2} + P_{r-1/2\ s-1/2}^{s-r+1} = P_{r-1/2\ s}^{s-r+1/2} + P_{rs}^{s-r+1}.$$

The $P_{h_B h_A}^{h_C}$ fulfil the equation.

Summary

- $$P^{h_C}_{h_B h_A} = \frac{1}{z^{2\eta h_B - 1}(1-z)^{2\eta h_C - 1}}, \quad h_C + h_B + h_A = \eta.$$

Summary

- $$P^{h_C}_{h_B h_A} = \frac{1}{z^{2\eta h_B - 1}(1-z)^{2\eta h_C - 1}}, \quad h_C + h_B + h_A = \eta.$$
- Asymptotic Freedom of Tensor-Gluons of spin s=1,2,.....

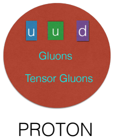
$$\beta_{GYM} = -\frac{\sum_{s=1}(12s^2-1)C_2(G) - 4n_f T(R)}{48\pi^2} g^3$$

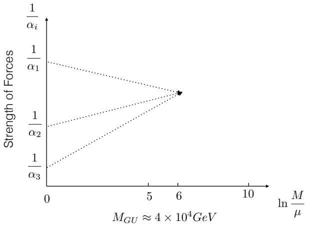
Summary

- $$P^{h_C}_{h_B h_A} = \frac{1}{z^{2\eta h_B - 1}(1-z)^{2\eta h_C - 1}}, \quad h_C + h_B + h_A = \eta.$$

- Asymptotic Freedom of Tensor-Gluons of spin s=1,2,.....

$$\beta_{GYM} = -\frac{\sum_{s=1}(12s^2-1)C_2(G) - 4n_f T(R)}{48\pi^2} g^3$$





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Thank You