Radiation spectra for h.d. SdS black holes: The effect of the temperature

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ΠΑΝΕΠΙΣΤΗΜΙΟ ΙΩΑΝΝΙΝΩΝ

Outline

- The gravitational background
- Black hole radiation
- The temperature definition
- Effective temperatures
- Power spectra

The gravitational background

Consider the gravitational action in $D = 4 + n$ dimensions

$$\mathcal{S}_{\mathcal{D}} = \int d^{4+n}x\sqrt{-g}\left(\frac{R_D}{2\kappa_D^2} - \Lambda\right) \tag{1}$$

The corresponding Einstein equations lead to the **h.d. Schwarzschild de-Sitter** solution in the bulk

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{h(r)} + r^2 d\Omega_{2+n}^2 \tag{2}$$

The induced metric on the brane is obtained by projection

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{h(r)} + r^2(d\theta^2 + d\phi^2 \sin^2\theta) \tag{3}$$

where the metric function in both cases is

$$h(r) = 1 - \frac{\mu}{r^{n+1}} - \tilde{\Lambda}r^2 \quad , \quad \tilde{\Lambda} := \frac{2\kappa_D^2\Lambda}{(n+2)(n+3)}. \tag{4}$$

Depending on the values of μ and $\tilde{\Lambda}$ the equation

$$h(r) = 1 - \frac{\mu}{r^{n+1}} - \tilde{\Lambda} r^2 = 0 \qquad (5)$$

will in general have **n+3** roots corresponding to the **horizons** of this spacetime. Constraining the parameter space in the following way [1]

$$\mu^2 \Lambda^{(n+1)} < \frac{4(n+1)^{(n+1)}}{(n+3)^{(n+3)}} \qquad (6)$$

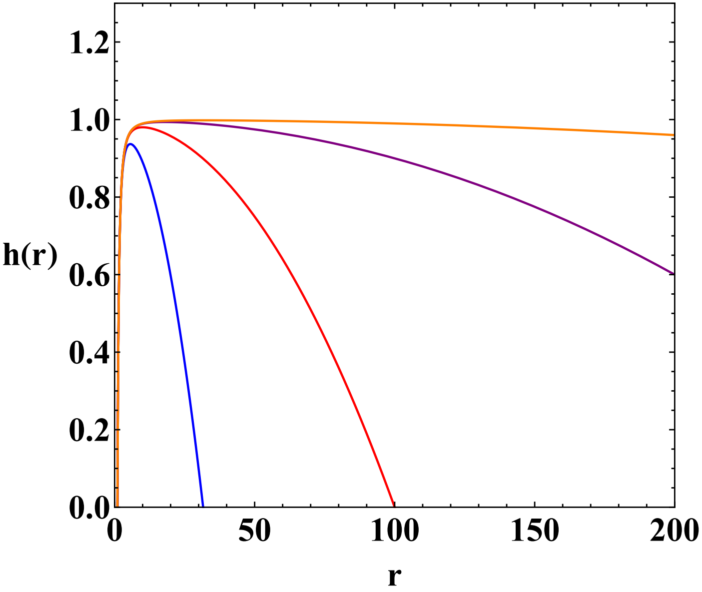
ensures that **only 2** real and positive horizons exist. The **black hole horizon** r_h and the **cosmological horizon** r_c where always

$$r_h \leqslant r_c. \qquad (7)$$

In the extreme case where the 2 horizons coincide we have the so called **Nariai limit** (black hole).

[1] C. Molina, Phys. Rev. D 68, 064007 (2003)

The global maximum of $h(r)$



We assume a free, massless scalar field Φ with a non-minimal coupling to gravity:

$$\mathcal{S} = -\frac{1}{2}\int d^{4+n}x\sqrt{-g}\left[\xi\Phi^2 R_D + g^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu}\right] \tag{8}$$

The Hawking radiation spectrum can be calculated by

$$\frac{d^2E}{dtd\omega} = \sum_l \frac{N_{l,n}}{2\pi}\frac{|A_{l,n}(\omega)|^2\omega}{exp(\omega/T_H)-1}\,,\quad N_{l,n} = \frac{(2l+n+1)(l+n)!}{l!(n+1)!}. \tag{9}$$

The function $|A_{l,n}(\omega)|^2$ is the **greybody factor** that modifies the blackbody radiation spectrum to that of a black hole.
Analytic[2] (valid only in the low-ω, low-Λ regime)
and **numerical**[3] (valid for all Λ and ω)
calculations have been performed for $|A_{l,n}(\omega)|^2$ in h.d. SdS.

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[2] P. Kanti, TP and N. Pappas, Phys. Rev. D 90, no. 12, 124077 (2014)

[3] TP, P. Kanti and N. Pappas, Phys. Rev. D 94, no. 2, 024035 (2016)

The black hole temperature

The standard definition of the temperature of the black hole is given in terms of the **surface gravity** [4]

$$T_i = \frac{\kappa_i}{2\pi} \quad , \quad \kappa_i^2 := -\frac{1}{2} \lim_{r \to r_i} K_{\mu;\nu} K^{\nu;\mu} \quad , \quad K = \gamma_t \frac{\partial}{\partial t} . \tag{10}$$

For spherically-symmetric gravitational backgrounds

$$\kappa_i = \frac{1}{2} \gamma_t \lim_{r \to r_i} |h(r)_{,r}| \tag{11}$$

The normalization constant γ_t for the asymptotically flat case ($\Lambda \to 0 \; , \; r_c \to \infty$) is taken to be

$$\gamma_t = 1 \quad , \quad K^\mu K_\mu = -1 \quad , \quad \lim_{r \to \infty} |h(r)| = 1 \tag{12}$$

4 J.M. Bardeen, B. Carter and S.W. Hawking, Comm. Math. Phys. 31 (1973) 161

Assuming $r_c \gg r_h$ each horizon can have its own independent thermodynamics[5] and so the corresponding temperatures are

$$\boxed{T_0 = \frac{\kappa_h}{2\pi} = \frac{1}{4\pi r_h}\left[(n+1) - (n+3)\tilde{\Lambda} r_h^2\right]} \tag{13}$$

$$T_c = -\frac{\kappa_c}{2\pi} = -\frac{1}{4\pi r_c}\left[(n+1) - (n+3)\tilde{\Lambda} r_c^2\right] \tag{14}$$

Bousso and Hawking proposed [6] an alternative normalization to account for the asymptotic non-flatness of SdS:

$$\boxed{T_{BH} = \frac{\kappa_h}{2\pi} = \frac{1}{4\pi r_h}\frac{1}{\sqrt{h(r_0)}}\left[(n+1) - (n+3)\tilde{\Lambda} r_h^2\right]} \tag{15}$$

At $r = r_0$ given by $\lim_{r\to r_0} h'(r) = 0$ there is a preferred non-accelerated observer where the gravitational attraction and Λ-repulsion cancel out.

$$\lim_{r\to r_0} |h(r)| \simeq 1 \tag{16}$$

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[5] D. Kastor and J. H. Traschen, Phys. Rev. D 47 (1993) 5370

[6] R. Bousso and S. W. Hawking, Phys. Rev. D 54 (1996) 6312

Effective temperatures

Another approaches have emerged during the recent years through **effective temperatures** involving both the black hole T_h and the cosmological horizon T_c temperatures.

A thermodynamical first law[7] for SdS black holes:

$$dM = TdS + PdV \tag{17}$$

with the mass of the black hole in the role of the enthalpy $M = -H$, $P = \Lambda/8\pi$ and $S = S_h + S_c$ leads to an effective temperature[8]:

$$\boxed{T_{eff-} = \left(\frac{1}{T_0} - \frac{1}{T_c}\right)^{-1} = \frac{T_0 T_c}{T_0 - T_c}} \tag{18}$$

Good news: In the limit $r_h \to 0 \quad , \quad T_{eff-} \to T_c$

Bad news: In the limit $r_c \to \infty \quad , \quad T_{eff-} \to 0 \Rightarrow$ Effective temperatures are invalid for $\Lambda = 0$

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7 J.M. Bardeen, B. Carter and S.W. Hawking, Comm. Math. Phys. 31 (1973) 161

8 M. Urano, A. Tomimatsu and H. Saida, Class. Quant. Grav. 26 (2009) 105010

Worse news: T_{eff-} exhibits some unphysical properties, especially for **RNdS** black holes $\rightarrow$ non-positive, infinite jumps at the critical point[9].

To circumvent these issues, an "**ad-hoc**" effective temperature was proposed[10] in analogy to T_{eff-} but under the assumption: $S = S_c - S_h$

$$T_{eff+} = \left(\frac{1}{T_0} + \frac{1}{T_c}\right)^{-1} = \frac{T_0 T_c}{T_0 + T_c} \tag{19}$$

Again for $r_h \rightarrow 0$, $T_{eff+} \rightarrow T_c$ and for $r_c \rightarrow \infty$, $T_{eff+} \rightarrow 0$.
Inspired by the above we propose[11] another expression for the effective temperature:

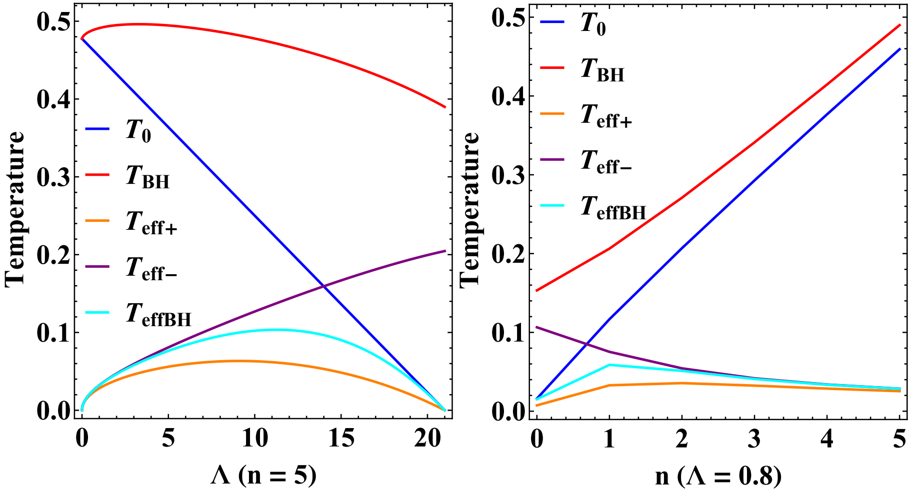
$$T_{effBH} = \left(\frac{1}{T_{BH}} - \frac{1}{T_c}\right)^{-1} = \frac{T_{BH} T_c}{T_{BH} - T_c} \tag{20}$$

9 For the lukewarm solution when $T_h = T_c$

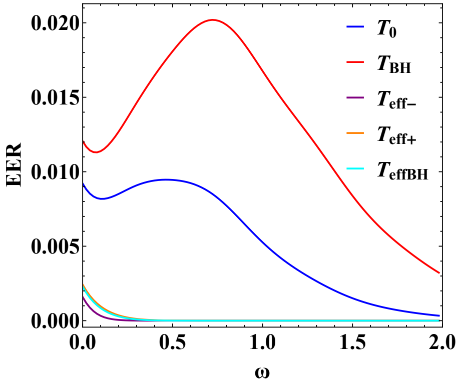
10 D. Kubiznak, R. B. Mann and M. Teo, Class. Quant. Grav. 34 (2017) no.6, 063001

11 In an upcoming paper that is to appear soon

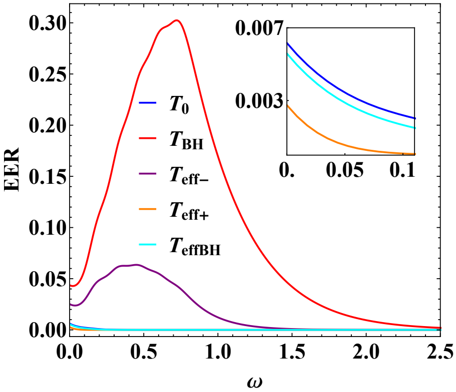
Temperatures as functions of Λ (in units of r_h^{-2}) and n



Power spectra on the brane - $\xi = 0$

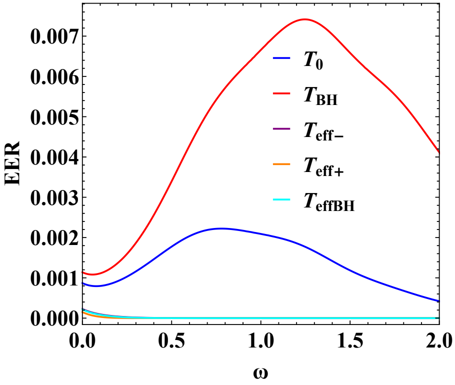


$n = 2\ ,\ \Lambda = 0.8$

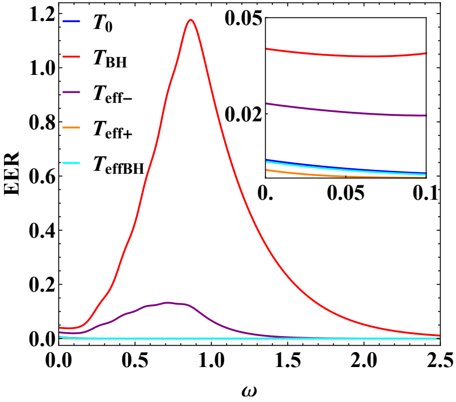


$n = 2\ ,\ \Lambda = 5$

Power spectra in the bulk - $\xi = 0$

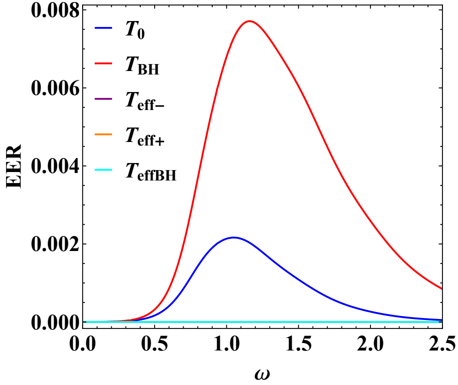


$n = 2\ , \Lambda = 0.8$

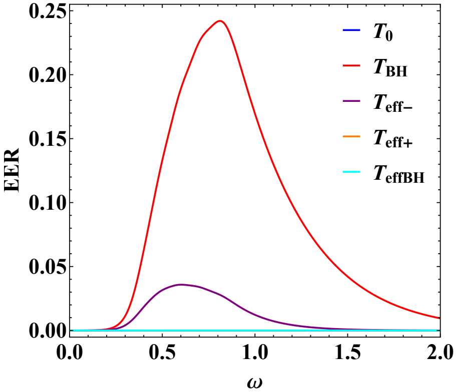


$n = 2\ , \Lambda = 5$

Power spectra on the brane - $\xi = 1$

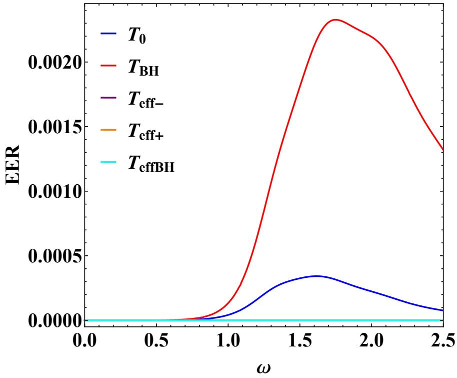


$n = 2\ ,\Lambda = 0.8$

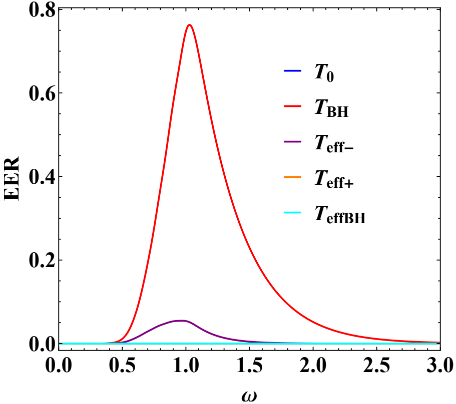


$n = 2\ ,\Lambda = 5$

Power spectra in the bulk - $\xi = 1$



$n = 2 \, , \Lambda = 0.8$



$n = 2 \, , \Lambda = 5$

Conclusions

- Various T_{eff} introduced to take in to account both horizons of SdS are valid only for $\Lambda \neq 0$ and lead in general to significantly suppressed EERs compared to the more "traditional" T_H definitions.
- T_{eff} get suppressed with n while T_0 and T_{BH} get enhanced.
- The effect of Λ varies. In the low-Λ regime T_0 and T_{BH} dominate while for the Nariai limit only T_{BH} and T_{eff-} assume non vanishing values.
- The inclusion of a non-minimal coupling (effective mass term for Φ) results in general suppression of the EERs and affects T_{eff} the most since the low-ω part of the spectrum is "flattened-out".
- In all cases studied here the radiation spectrum for an SdS black hole with T_{BH} is the most dominant one both in the bulk and on the brane.
- What about the total emissivity on the brane and in the bulk? (in progress)

Thank you!