Radiation spectra for h.d. SdS black holes: The effect of the temperature

Thomas Pappas

Theory Division
Physics Department
University of Ioannina

TP and Kanti Panagiota
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Outline

- The gravitational background
- Black hole radiation
- The temperature definition
- Effective temperatures
- Power spectra
The gravitational background

Consider the gravitational action in \( D = 4 + n \) dimensions

\[
S_D = \int d^{4+n}x \sqrt{-g} \left( \frac{R_D}{2\kappa_D^2} - \Lambda \right)
\]  

(1)

The corresponding Einstein equations lead to the \textbf{h.d. Schwarzschild de-Sitter} solution in the bulk

\[
ds^2 = -h(r)dt^2 + \frac{dr^2}{h(r)} + r^2d\Omega_{2+n}^2
\]  

(2)

The induced metric on the brane is obtained by projection

\[
ds^2 = -h(r)dt^2 + \frac{dr^2}{h(r)} + r^2(d\theta^2 + d\phi^2 \sin^2 \theta)
\]  

(3)

where the metric function in both cases is

\[
h(r) = 1 - \frac{\mu}{r^{n+1}} - \tilde{\Lambda}r^2,
\tilde{\Lambda} := \frac{2\kappa_D^2\Lambda}{(n + 2)(n + 3)}.
\]  

(4)
Depending on the values of $\mu$ and $\tilde{\Lambda}$ the equation

$$h(r) = 1 - \frac{\mu}{r^{n+1}} - \tilde{\Lambda}r^2 = 0$$

will in general have $n+3$ roots corresponding to the horizons of this spacetime. Constraining the parameter space in the following way $^1$

$$\mu^2 \Lambda^{(n+1)} < \frac{4(n + 1)(n+1)}{(n + 3)(n+3)}$$

ensures that only 2 real and positive horizons exist. The black hole horizon $r_h$ and the cosmological horizon $r_c$ where always

$$r_h \leq r_c.$$ 

In the extreme case where the 2 horizons coincide we have the so called Nariai limit (black hole).

The global maximum of $h(r)$
We assume a free, massless scalar field $\Phi$ with a non-minimal coupling to gravity:

$$S = -\frac{1}{2} \int d^{4+n}x \sqrt{-g} \left[ \xi \Phi^2 R_D + g^{\mu\nu} \Phi,_{\mu} \Phi,_{\nu} \right]$$

(8)

The Hawking radiation spectrum can be calculated by

$$\frac{d^2 E}{dt d\omega} = \sum_l \left( \frac{N_{l,n}}{2\pi} \frac{|A_{l,n}(\omega)|^2 \omega}{\exp(\omega/T_H) - 1} \right), \quad N_{l,n} = \frac{(2l + n + 1)(l + n)!}{l!(n + 1)!}.$$  

(9)

The function $|A_{l,n}(\omega)|^2$ is the greybody factor that modifies the blackbody radiation spectrum to that of a black hole.

**Analytic**$^2$ (valid only in the low-$\omega$, low-$\Lambda$ regime) and **numerical**$^3$ (valid for all $\Lambda$ and $\omega$) calculations have been performed for $|A_{l,n}(\omega)|^2$ in h.d. SdS.

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$^2$ P. Kanti, TP and N. Pappas, Phys. Rev. D 90, no. 12, 124077 (2014)

The black hole temperature

The standard definition of the temperature of the black hole is given in terms of the surface gravity

\[ T_i = \frac{\kappa_i}{2\pi} , \quad \kappa_i^2 := -\frac{1}{2} \lim_{r \to r_i} K_{\mu;\nu} K^{\nu;\mu} , \quad K = \gamma_t \frac{\partial}{\partial t}. \]  

(10)

For spherically-symmetric gravitational backgrounds

\[ \kappa_i = \frac{1}{2} \gamma_t \lim_{r \to r_i} |h(r),r| \]  

(11)

The normalization constant \( \gamma_t \) for the asymptotically flat case \((\Lambda \to 0 , \ r_c \to \infty)\) is taken to be

\[ \gamma_t = 1 , \quad K^{\mu}K_\mu = -1 , \quad \lim_{r \to \infty} |h(r)| = 1 \]  

(12)

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Assuming $r_c \gg r_h$ each horizon can have its own independent thermodynamics\(^5\) and so the corresponding temperatures are

\[
T_0 = \frac{\kappa_h}{2\pi} = \frac{1}{4\pi r_h} \left[ (n + 1) - (n + 3)\tilde{\Lambda} r_h^2 \right]
\]  
\[
(13)
\]

\[
T_c = -\frac{\kappa_c}{2\pi} = -\frac{1}{4\pi r_c} \left[ (n + 1) - (n + 3)\tilde{\Lambda} r_c^2 \right]
\]  
\[
(14)
\]

Bousso and Hawking proposed\(^6\) an alternative normalization to account for the asymptotic non-flatness of SdS:

\[
T_{BH} = \frac{\kappa_h}{2\pi} = \frac{1}{4\pi r_h} \frac{1}{\sqrt{h(r_0)}} \left[ (n + 1) - (n + 3)\tilde{\Lambda} r_h^2 \right]
\]  
\[
(15)
\]

At $r = r_0$ given by $\lim_{r \to r_0} h'(r) = 0$ there is a preferred non-accelerated observer where the gravitational attraction and $\Lambda$-repulsion cancel out.

\[
\lim_{r \to r_0} |h(r)| \simeq 1
\]  
\[
(16)
\]

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\(^6\) R. Bousso and S. W. Hawking, Phys. Rev. D 54 (1996) 6312
Effective temperatures

Another approaches have emerged during the recent years through effective temperatures involving both the black hole $T_h$ and the cosmological horizon $T_c$ temperatures.

A thermodynamical first law\(^7\) for SdS black holes:

$$dM = TdS + PdV$$  \hspace{1cm} (17)

with the mass of the black hole in the role of the enthalpy $M = -H$, $P = \Lambda / 8\pi$ and $S = S_h + S_c$ leads to an effective temperature\(^8\):

$$T_{\text{eff}} = \left( \frac{1}{T_0} - \frac{1}{T_c} \right)^{-1} = \frac{T_0 T_c}{T_0 - T_c}$$  \hspace{1cm} (18)

**Good news**: In the limit $r_h \rightarrow 0$, $T_{\text{eff}} \rightarrow T_c$.

**Bad news**: In the limit $r_c \rightarrow \infty$, $T_{\text{eff}} \rightarrow 0 \Rightarrow$ Effective temperatures are invalid for $\Lambda = 0$.

\(7\) J.M. Bardeen, B. Carter and S.W. Hawking, Comm. Math. Phys. 31 (1973) 161

\(8\) M. Urano, A. Tomimatsu and H. Saida, Class. Quant. Grav. 26 (2009) 105010
Worse news: $T_{eff-}$ exhibits some unphysical properties, especially for RNdS black holes → non-positive, infinite jumps at the critical point\(^9\).

To circumvent these issues, an "ad-hoc" effective temperature was proposed\(^{10}\) in analogy to $T_{eff-}$ but under the assumption: $S = S_c - S_h$

\[
T_{eff+} = \left( \frac{1}{T_0} + \frac{1}{T_c} \right)^{-1} = \frac{T_0 T_c}{T_0 + T_c}
\]

(19)

Again for $r_h \to 0$ , $T_{eff+} \to T_c$ and for $r_c \to \infty$ , $T_{eff+} \to 0$.

Inspired by the above we propose\(^{11}\) another expression for the effective temperature:

\[
T_{effBH} = \left( \frac{1}{T_{BH}} - \frac{1}{T_c} \right)^{-1} = \frac{T_{BH} T_c}{T_{BH} - T_c}
\]

(20)

\(^9\) For the lukewarm solution when $T_h = T_c$

\(^{10}\) D. Kubiznak, R. B. Mann and M. Teo, Class. Quant. Grav. 34 (2017) no.6, 063001

\(^{11}\) In an upcoming paper that is to appear soon
Temperatures as functions of $\Lambda$ (in units of $r^{-2}_h$) and $n$.
Power spectra on the brane - $\xi = 0$

$n = 2, \Lambda = 0.8$

$n = 2, \Lambda = 5$
Power spectra in the bulk - $\xi = 0$

$n = 2, \Lambda = 0.8$

$n = 2, \Lambda = 5$
Power spectra on the brane - $\xi = 1$

$n = 2$, $\Lambda = 0.8$

$n = 2$, $\Lambda = 5$
Power spectra in the bulk - $\xi = 1$

$n = 2, \Lambda = 0.8$

$n = 2, \Lambda = 5$
Conclusions

- Various $T_{eff}$ introduced to take into account both horizons of SdS are valid only for $\Lambda \neq 0$ and lead in general to significantly suppressed EERs compared to the more "traditional" $T_H$ definitions.

- $T_{eff}$ get suppressed with $n$ while $T_0$ and $T_{BH}$ get enhanced.

- The effect of $\Lambda$ varies. In the low-$\Lambda$ regime $T_0$ and $T_{BH}$ dominate while for the Nariai limit only $T_{BH}$ and $T_{eff}$ assume non-vanishing values.

- The inclusion of a non-minimal coupling (effective mass term for $\Phi$) results in general suppression of the EERs and affects $T_{eff}$ the most since the low-$\omega$ part of the spectrum is "flattened-out".

- In all cases studied here the radiation spectrum for an SdS black hole with $T_{BH}$ is the most dominant one both in the bulk and on the brane.

- What about the total emissivity on the brane and in the bulk? (in progress)
Thank you!