$\mathcal{R}$-parity violation in $\mathcal{F}$-theory

Thanos Karozas

Physics Department,
University of Ioannina.

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Outline

$\mathcal{R}$-Part:

$\mathcal{R}$-Parity, RPV-MSSM, Proton Decay, Motivation

$\mathcal{F}$-Part:

$\mathcal{F}$-theory, $SO(12)$ point, Plots & Numerics, Summary

M. C. Romão, AK, S. F. King, G.K. Leontaris, A. K. Meadowcroft: 10.1007/*JHEP*11(2016)081

$\mathcal{R}$-Part

Intro & Motivation

$\mathcal{R}$-parity

⋆ MSSM -$\mathcal{R}$ parity violation (RPV or $\not{\mathcal{R}}$) superpotential:

$$W_{RPV} = \underbrace{\mu_i H_u L_i + \frac{1}{2}\lambda_{ijk} L_i L_j e_k^c + \lambda'_{ijk} L_i Q_j d_k^c}_{\text{L violation}} + \overbrace{\frac{1}{2}\lambda''_{ijk} u_i^c d_j^c d_k^c}^{\text{B violation}}$$

⋆ Add a new discrete symmetry to eliminate these terms, called "$\mathcal{R}$-parity" (Farrat & Fayet, Phys. Lett. 76B (1978) 575–579.)

$$\boxed{P_R = (-1)^{3(B-L)+2s}}$$

- $P_R = +1$ for Standard Model (SM) particles.
- $P_R = -1$ for SUSY particles.

Matter parity

$$W_{RPV} = \underbrace{\mu_i H_u L_i + \frac{1}{2}\lambda_{ijk} L_i L_j e^c_k + \lambda'_{ijk} L_i Q_j d^c_k}_{\text{L violation}} + \overbrace{\frac{1}{2}\lambda''_{ijk} u^c_i d^c_j d^c_k}^{\text{B violation}}$$

⋆ An alternative symmetry with the same physical results is "Matter parity" :

- $(L_i, e^c_i, Q_i, u^c_i, d^c_i) \rightarrow P_M = -1$
- $(H_u, H_d) \rightarrow P_M = +1$

⋆ **This forbids all terms with an odd power of matter fields and thus forbids all the terms in W_{RPV}.**

RPV-SUSY

⋆ Plethora of new couplings (=48), provide a rich phenomenology :

- Single s-particle production is allowed.
- LSP is unstable (decays to leptons or jets)

(H. Dreiner et al: 1205.0557, Review: R.Barbier et al hep-ph/0406039, LHC-Run I Review: A. Redelbach, arXiv:1512.05956)

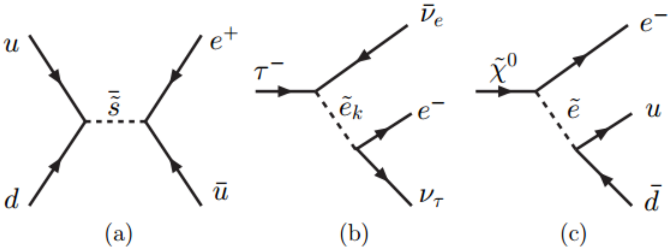


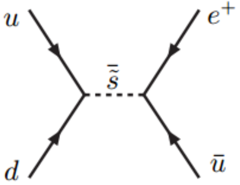
Figure : Examples of RPV processes: (a) Proton Decay via λ''_{112} and λ'_{112}, (b) Tau decay via two λ_{13k} insertions, (c) Neutralino decay via λ'_{111}.

Proton decay

⋆ Proton decay (PD) requires both $\not{L}$ and $\not{B}$.

- $\Gamma(p \to \pi^0 e^+) \sim |\lambda'_{112}\lambda''_{112}|^2 \frac{m^5_{proton}}{\tilde{m}^4_{s_R}} < \frac{1}{10^{33} yr} \Rightarrow$

  $|\lambda'_{112}\lambda''_{112}| < 5 \times 10^{-27} \left(\frac{\tilde{m}_{s_R}}{1\,TeV}\right)^2.$



⋆ Very strict bound $\longmapsto$ at least one of the couplings is zero.

- **Only B conservation** $\longmapsto$ $\not{L}MSSM$
- **Only L conservation** $\longmapsto$ $\not{B}MSSM$

(Dimopoulos et al, doi:10.1016/0370-2693(88)91418-9)

⋆ **Baryon-parity and lepton-parity are two possible solutions to maintain a stable proton and allow for RPV.**

Example $\longrightarrow$

Motivation

(AK, S.F.King, G.K.Leontaris, A.K.Meadowcroft, *doi* : 10.1007/*JHEP*10(2015)041)

Low Energy Spectrum	D_4 rep	$U(1)_{t_5}$	Z_2
Q_3, u_3^c, e_3^c	1_{+-}	0	$-$
u_2^c	1_{++}	1	$+$
u_1^c	1_{++}	0	$+$
$Q_{1,2}, e_{1,2}^c$	2	0	$-$
L_i, d_i^c	1_{+-}	0	$-$
ν_3^c	1_{+-}	0	$-$
$\nu_{1,2}^c$	2	0	$-$
H_u	1_{++}	0	$+$
H_d	1_{++}	-1	$+$

Table : Low energy spectrum of a $SU(5) \times D_4 \times U(1)$ F-theory inspired model with a geometric parity. The fields $u_{1,2}^c$ have different assignment in comparison with the conventional matter parity. As a result $\not{B}$ terms: $u_1^c d_j^c d_k^c \longmapsto$ neutron-antineutron oscillations (Goity & Sher)

RPV in F-theory?

$\star$ So far in F-theory...plethora of works on $SU(5)$ Yukawa couplings
(Vafa et al, Ibanez & Font, Hayashi et al, Leontaris & Ross, Palti et al, Marchesano et al....)

$$10 \times 10 \times 5_H \to y_{top} \quad \surd$$

$$10 \times \bar{5}_M \times \bar{5}_H \to y_{bottom}, \quad y_{tau} \quad \surd$$

$\star$ What about RPV couplings...

$$10 \times \bar{5}_M \times \bar{5}'_M \to y_{RPV} \qquad ...?$$

$\star$ A first estimation: *we expect a similar behavior to* y_{bottom} *coupling* .

$\mathcal{F}$-Part

$\mathcal{F}$-theory, the $SO(12)$ point and RPV

M. C. Romão, AK, S. F. King, G.K. Leontaris, A. K. Meadowcroft: 10.1007/*JHEP*11(2016)081

$\mathcal{F}$-theory (Basic)

⋆ **Geometrisation of Type II-B superstring** (Vafa 1996)

II-B: closed string spectrum obtained by combining left and right moving open strings with NS and R-boundary conditions.

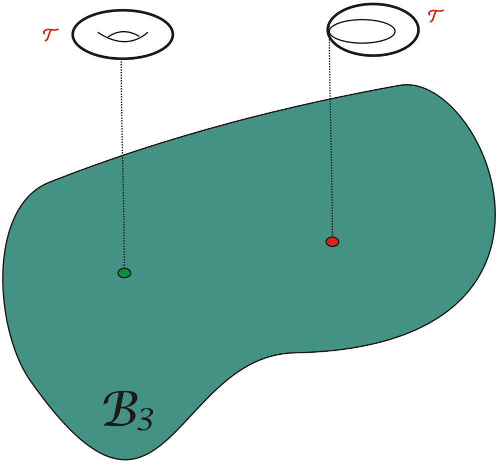
⋆ **Geometrical Picture**:

- Take the 6-d compact space to be CY 3-fold base B_3.

- Associate a torus $\tau = C_0 + \imath/g_s$ at each point of B_3.

$\Longrightarrow$ Internal space elliptically fibered **CY** 4-fold $\mathcal{X}$ over B_3

$\hookrightarrow$ F-theory defined on the background $\mathcal{R}^{3,1} \times \mathcal{X}$ $\hookleftarrow$



Red points: pinched torus $\longmapsto$ 7-branes $\perp$ B_3.

Singularities

$\star$ Fibration is described by the Weierstraß Equation

$$y^2 = x^3 + f(z)\,x + g(z) \tag{1}$$

x, y parameters of the fibration.
$f(z)$, $g(z) \longmapsto 8\,\&\,12$ degree polynomials in z.

$\star\star$ For each point of B_3, eq(1) describes a torus labeled by z.

$\star\star\star$ The fiber degenerates at the zeros of the discriminant

$$\Delta = 4\,f^3 + 27\,g^2$$

$$\Downarrow$$

$\Delta = 0 \implies$ singularity of internal mainfold

Singularities & Gauge Symmetry

⋆ Type of Manifold singularity is specified by the vanishing order of f(z), g(z) polynomials

⋆⋆ Singularities are classified in terms of $\mathcal{ADE}$ Lie groups. (Kodaira 1968)

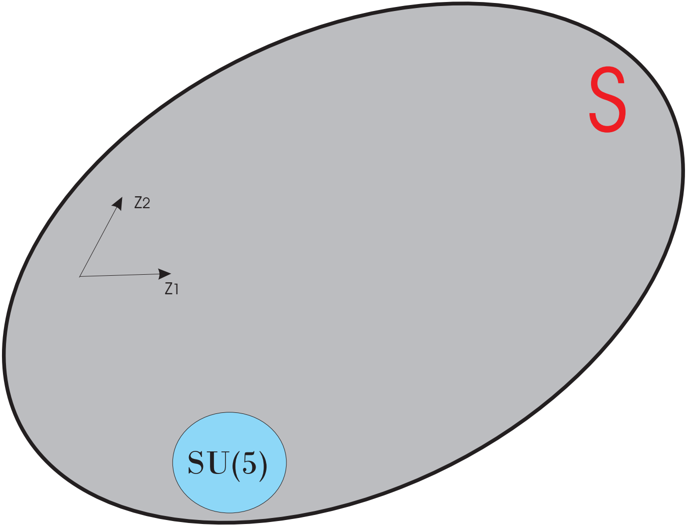
$$\boxed{\mathcal{X}\text{-Singularities} \leftrightarrows \text{Gauge Symmetry}}$$

⋆ ⋆ ⋆ The maximum symmetry enhancement is E_8,

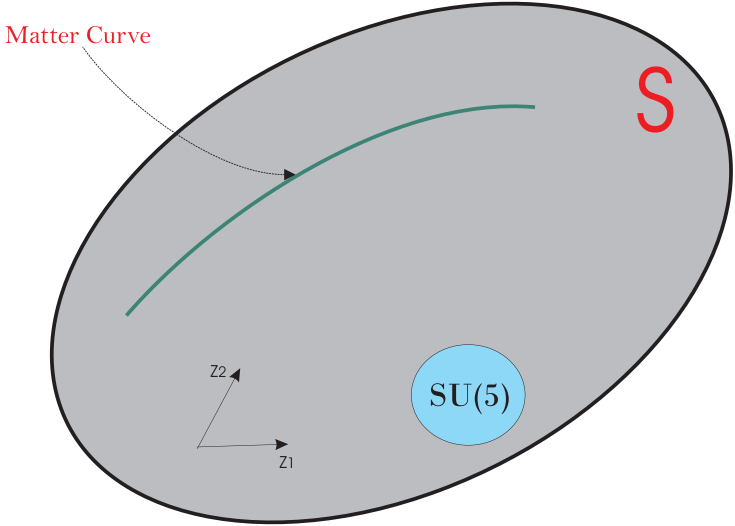
$$E_8 \to \mathcal{G}_{GUT} \times SU(n)_\perp$$

with $\mathcal{G}_{GUT} = E_6, SO(10), SU(5)$ for $n = 3, 4, 5$.

⋆ in F-theory: 7-branes wrap certain class of *'internal'* **2-complex dim.** surface **S** associated to gauge group $\mathcal{G}_S$ (here taken to be $SU(5)$)

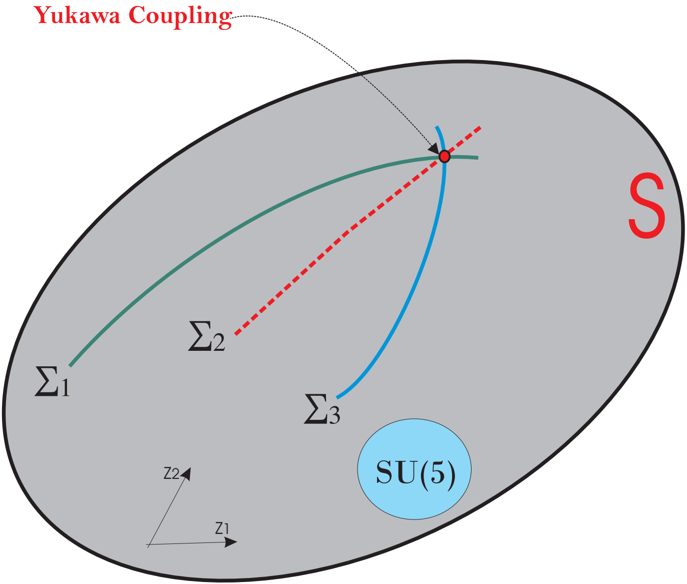


Matter resides along intersections with other 7-branes...



Along a matter curve Σ gauge symmetry is enhanced...

Yukawa couplings at Triple intersections...



gauge symmetry ... further ... enhanced!

SU(5) :Singularity enhancement

⋆ Matter curves accommodating $\bar{\mathbf{5}}$ are associated with $SU(6)$

$$\Sigma_{\bar{5}} = S \cap S_{\bar{5}} \Rightarrow \qquad SU(5) \to SU(6)$$
$$\mathrm{ad}_{SU_6} = 35 \to \qquad 24_0 + 1_0 + 5_6 + \bar{5}_{-6}$$

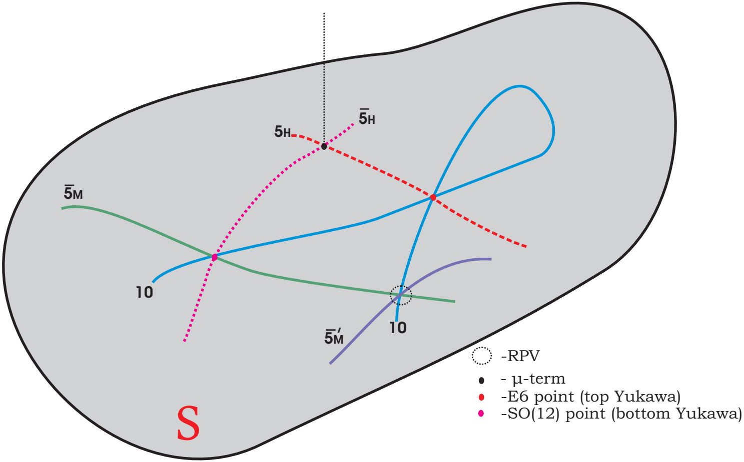
⋆ Matter curves accommodating **10** are associated with $SO(10)$

$$\Sigma_{10} = S \cap S_{10} \Rightarrow \qquad SU(5) \to SO(10)$$
$$\mathrm{ad}_{SO_{10}} = 45 \to \qquad 24_0 + 1_0 + 10_4 + \overline{10}_{-4}$$

⋆ Further enhancement in triple intersections→ **Yukawas:**

$$SO(10) \equiv E_5 \Rrightarrow \qquad E_6 \to \mathbf{10} \times \mathbf{10} \times \mathbf{5}$$
$$SU(6) \Rrightarrow \qquad SO(12) \to \mathbf{10} \times \bar{\mathbf{5}} \times \bar{\mathbf{5}}$$

$\Longrightarrow$ **RPV couplings** $\longrightarrow$ **SO(12) point enhancement**


5H
5̄H
5̄M
10
5̄′M
10
S
-RPV
- μ-term
-E6 point (top Yukawa)
-SO(12) point (bottom Yukawa)

Effective theory

$\star$ The 4-d theory can be obtained by integrating out the 8-d theory over S

$$W = m_*^4 \int_S \mathrm{Tr}(F \wedge \Phi)$$

- $F = dA - iA \wedge A$ is the field-strength of the gauge vector boson A .
- Φ is $(2,0)$-form on S.
- m_* : F-theory characteristic scale

$\star\star$ Away from the enh. point Φ breaks $SO(12) \to$ GUT group $SU(5)$:

$$SO(12) \to SU(5) \times U(1) \times U(1)$$

Fluxes

⋆ We also need fluxes

- $\langle F \rangle \rightarrow$ chirality on the matter curves
- $\langle F_Y \rangle \rightarrow$ breaks the GUT down to SM

⋆⋆ Collectively the total flux is:

$$\begin{aligned}\langle F_{total} \rangle =& i(dz_2 \wedge d\bar{z}_2 - dz_1 \wedge d\bar{z}_1)Q_P \\ &+ i(dz_1 \wedge d\bar{z}_2 + dz_2 \wedge d\bar{z}_1)Q_S \\ &+ i(dz_2 \wedge d\bar{z}_2 + dz_1 \wedge d\bar{z}_1)M_{z_1 z_2} Q_F \end{aligned} \tag{2}$$

with the definitions

$$Q_P = M Q_F + \tilde{N}_Y Q_Y \tag{3}$$

$$Q_S = N_a Q_{z_1} + N_b Q_{z_2} + N_Y Q_Y \tag{4}$$

Sector	SM	q_F	q_{z_1}	q_{z_2}	q_S	q_P
a_1	$(\bar{\mathbf{3}},\mathbf{1})_{-\frac{1}{3}}$	1	-1	0	$-N_a-\frac{1}{3}N_Y$	$M-\frac{1}{3}\tilde{N}_Y$
a_2	$(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	1	-1	0	$-N_a+\frac{1}{2}N_Y$	$M+\frac{1}{2}\tilde{N}_Y$
b_1	$(\bar{\mathbf{3}},\mathbf{1})_{\frac{2}{3}}$	-1	0	1	$N_b+\frac{2}{3}N_Y$	$-M+\frac{2}{3}\tilde{N}_Y$
b_2	$(\mathbf{3},\mathbf{2})_{-\frac{1}{6}}$	-1	0	1	$N_b-\frac{1}{6}N_Y$	$-M-\frac{1}{6}\tilde{N}_Y$
b_3	$(\mathbf{1},\mathbf{1})_{-1}$	-1	0	1	N_b-N_Y	$-M-\tilde{N}_Y$
c_1	$(\bar{\mathbf{3}},\mathbf{1})_{-\frac{1}{3}}$	0	1	-1	$N_a-N_b-\frac{1}{3}N_Y$	$-\frac{1}{3}\tilde{N}_Y$
c_2	$(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	0	1	-1	$N_a-N_b+\frac{1}{2}N_Y$	$\frac{1}{2}\tilde{N}_Y$

Table : Complete data of sectors present in the three curves crossing in an $SO(12)$ enhancement point considering the effects of non-vanishing fluxes.

Coupling coefficients

$\star$ Matter fields arise as fluctuations of the 8-dim fields

$$\Psi_{8D} = \phi_{4D} \times \psi_{int}$$

$\star\star$ Operator coefficients arise as overlaps of wavefunctions

$$\int_{8D} \Psi_1 \Psi_2 \Psi_3 = \int_{4D} \phi_1 \phi_2 \phi_3 \left(\int_S \psi_1 \psi_2 \psi_3 \right)$$

$\star\star\star$ Solve the eom for the zero mode wavefunctions (Font et al, 2012)

(Heckman et al, 2008)

Wavefunctions

⋆ Wavefunctions (WF) in holomorphic gauge:

$$\vec{\psi}^{(b)hol}_{10_M} = \vec{v}^{(b)}\chi^{(b)hol}_{10_M} = \vec{v}^{(b)}\kappa^{(b)}_{10_M}\, e^{\lambda_b z_2(\bar{z}_2-\zeta_b\bar{z}_1)}$$
$$\vec{\psi}^{(a)hol}_{5_M} = \vec{v}^{(a)}\chi^{(a)hol}_{5_M} = \vec{v}^{(a)}\kappa^{(a)}_{5_M} e^{\lambda_a z_1(\bar{z}_1-\zeta_a\bar{z}_2)}$$
$$\vec{\psi}^{(c)hol}_{5_H} = \vec{v}^{(c)}\chi^{(c)hol}_{5_H} = \vec{v}^{(c)}\kappa^{(c)}_{5_H} e^{(z_1-z_2)(\zeta_c\bar{z}_1-(\lambda_c-\zeta_c)\bar{z}_2)}$$
$$\vec{\psi}^{(c)hol}_{5_M} = \vec{v}^{(c)}\chi^{(c)hol}_{5_M} = \vec{v}^{(c)}\kappa^{(c)}_{5_M} e^{(z_1-z_2)(\zeta_c\bar{z}_1-(\lambda_c-\zeta_c)\bar{z}_2)}.$$

where λ_ρ is the smallest eigenvalue of the matrix

$$m_\rho = \begin{pmatrix} -q_P & q_S & im^2 q_{z_1} \\ q_S & q_P & im^2 q_{z_2} \\ -im^2 q_{z_1} & -im^2 q_{z_2} & 0 \end{pmatrix}. \quad (5)$$

b,τ and RPV couplings

$\star$ bottom /tau Yukawa:

$$\boxed{y_{b,\tau} = \pi^2 \left(\frac{m_*}{m}\right)^4 t_{abc}\kappa^{(b)}_{10_M}\kappa^{(a)}_{5_M}\kappa^{(c)}_{5_H}} \tag{6}$$

$\star\star$ RPV coupling:

$$\boxed{y_{RPV} = \pi^2 \left(\frac{m_*}{m}\right)^4 t_{abc}\kappa^{(b)}_{10_M}\kappa^{(a)}_{5_M}\kappa^{(c)}_{5_M}} \tag{7}$$

As we observe the flux dependence is hidden on the **normalization factors.**

Normalization factors

$\star$ fixed by imposing canonical kinetic terms

$$1 = 2m_*^4 ||\vec{v}^{(e)}||^2 \int (\chi^{(e)})_i^* \chi_i^{(e)} \mathrm{dVol}_S$$

$\star\star$ partial results...

$$|\kappa_{10_M}^{(b)}|^2 = -4\pi g_s \sigma^2 \cdot \frac{q_P(b)(-2\lambda_b + q_P(b)(1+\zeta_b^2))}{\lambda_b(1+\zeta_b^2) + m^4}$$

$$|\kappa_{5_H}^{(c)}|^2 = -4\pi g_s \sigma^2 \cdot \frac{2(q_P(c)+\zeta_c)(q_P(c)+2\zeta_c - 2\lambda_c) + (q_S(c)+\lambda_c)^2}{\zeta_c^2 + (\lambda_c - \zeta_c)^2 + m^4},$$

where $\sigma = (m/m_{st})^2$, with m_{st} the string scale .

Numerical analysis

$\star$ The couplings can be written as :

$$y_{b,\tau} = 2g_s^{1/2}\sigma \; y'_{b,\tau}$$
$$y_{RPV} = 2g_s^{1/2}\sigma \; y'_{RPV}$$

$\star\star$ five parameters - N_a, N_b, M, N_Y and $\tilde{N}_Y$

$\star\star\star$ constraint: elimination of Higgs colour triplets $\Longrightarrow$ (Font & Ibanez et al.)

$$N_b = N_a - \frac{1}{3}N_Y$$

$\star\star\star\star$ At the GUT scale $Y_\tau/Y_b = 1.37 \pm 0.1 \pm 0.2$ (G.Ross & M. Serna 2008)

Y_τ / Y_b

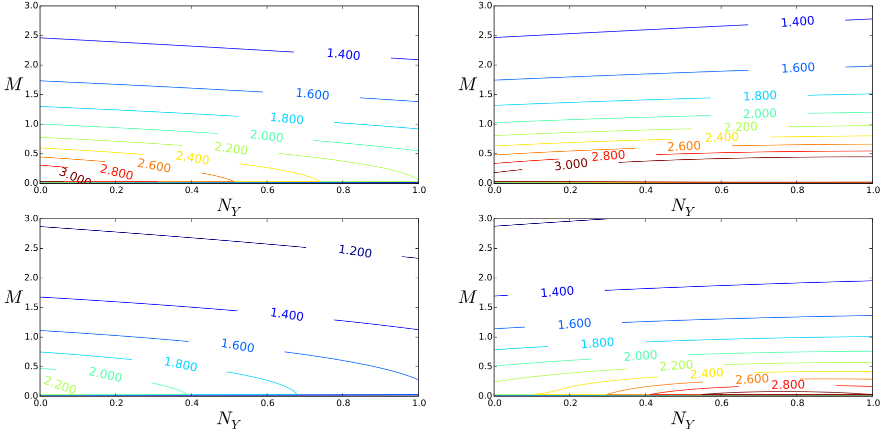


Figure : Ratio between bottom Yukawa and tau Yukawa couplings, shown as contours in the plane of local fluxes. The requirement for chiral matter and absence of coloured Higgs triplets fixes $N_b = N_a - \frac{1}{3}N_Y$

RPV (in absence of N_Y and $\tilde{N}_Y$)

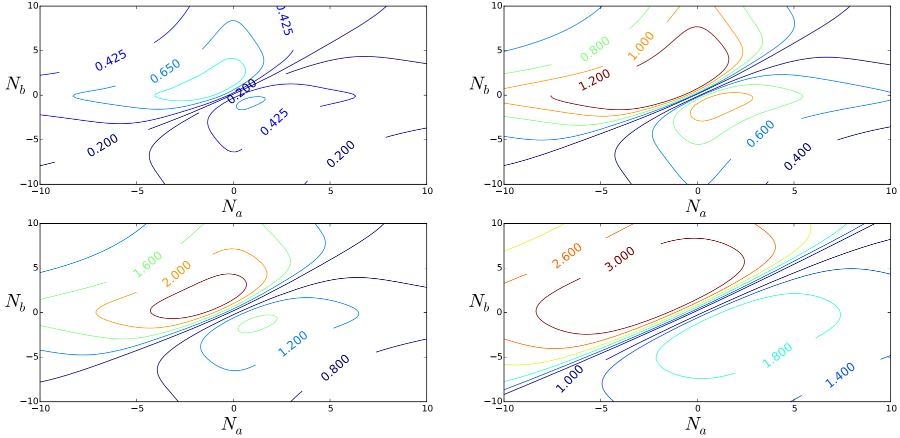
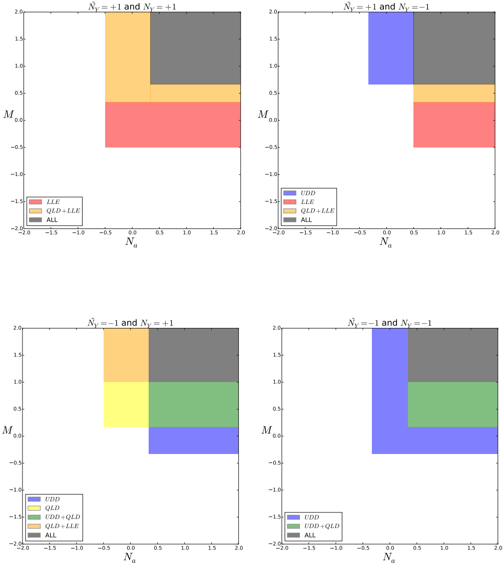


Figure : Dependency of the RPV coupling (in units of $2g_s^{1/2}\sigma$) on the (N_a, N_b)-plane, in absence of hypercharge fluxes and for different values of M. Top: left $M = 0.5$, right $M = 1.0$. Bottom: left $M = 2.0$, right $M = 3.0$.

RPV allowed regions



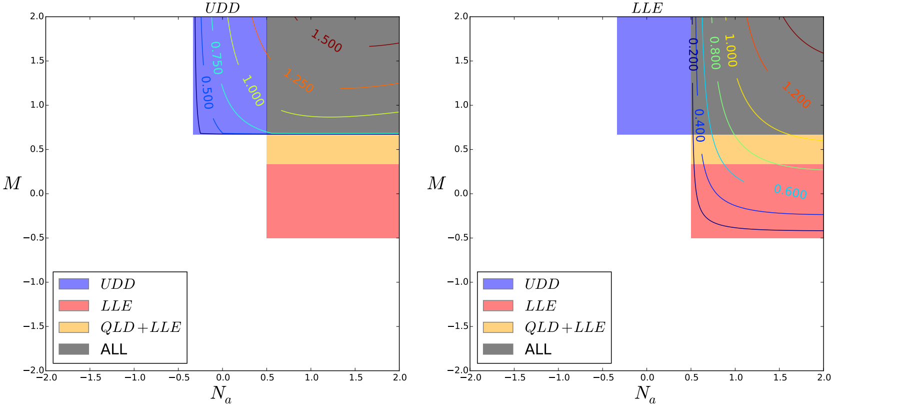


Figure : Allowed regions in the parameter space for different RPV couplings with $\tilde{N}_Y = -N_Y = 1$. We have also include the corresponding contours for the $u^c d^c d^c$ operator (left) and LLe^c (right).

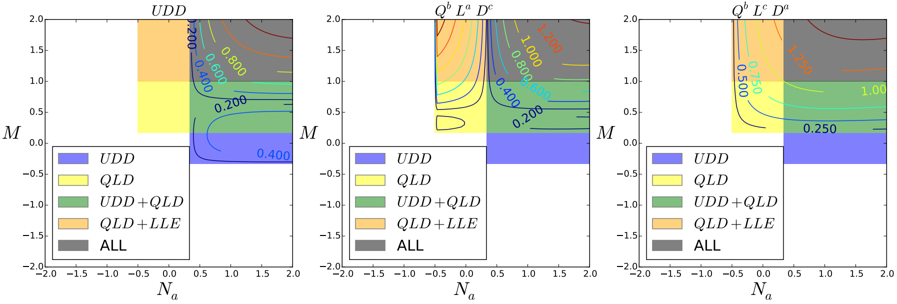


Figure : Allowed regions in the parameter space for different RPV couplings with $N_Y = -\tilde{N}_Y = 1$. We have also include the corresponding contours for the $u^c d^c d^c$ operator (left) and QLd^c (middle and right). The scripts a, b and c refer to which sector each state 'lives'.

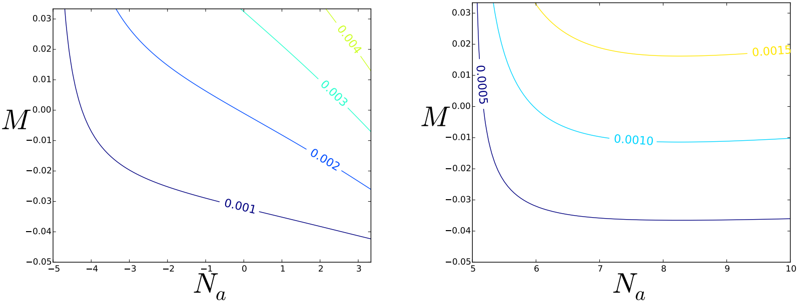
Bounds

⋆ partial results in (Allanach, Dedes & Dreiner 1999)

ijk	λ_{ijk}	λ'_{ijk}	λ''_{ijk}
111	-	1.5×10^{-4}	-
112	-	6.7×10^{-4}	4.1×10^{-10}
113	-	0.0059	1.1×10^{-8}
121	0.032	0.0015	4.1×10^{-10}
122	0.032	0.0015	-
123	0.032	0.012	1.3×10^{-7}
131	0.041	0.0027	1.1×10^{-8}
132	0.041	0.0027	1.3×10^{-7}
133	0.0039	4.4×10^{-4}	-
211	0.032	0.0015	-
212	0.032	0.0015	(1.23)
213	0.032	0.016	(1.23)
231	0.046	0.0027	(1.23)
232	0.046	0.0028	(1.23)
233	0.046	0.048	-
311	0.041	0.0015	-
312	0.041	0.0015	0.099
313	0.0039	0.0031	0.015
321	0.046	0.0015	0.099
322	0.046	0.0015	-
333	-	0.091	-

y_{RPV}

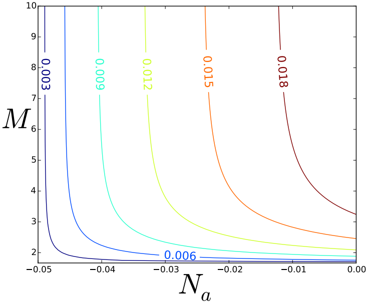
⋆ For $tan\beta = 5$, $y_b(M_{GUT}) \simeq 0.03$.



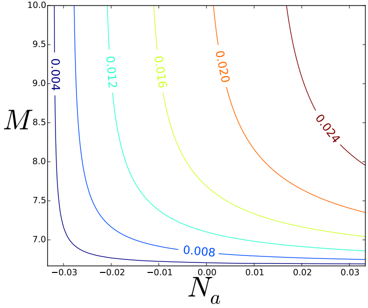
(a) λLLe^c region with $N_Y = 10$, $\tilde{N}_Y = 0.1$

(b) λLLe^c region with $N_Y = -10$, $\tilde{N}_Y = 0.1$

y_{RPV}



(c) $\lambda' Q L d^c$ region with $N_Y = 0.1$, $\tilde{N}_Y = -10$



(d) $\lambda'' u^c d^c d^c$ region with $N_Y = -0.1$, $\tilde{N}_Y = -10$

Summary

* $\mathcal{R}$- parity violation (RPV) in semi-local $\mathcal{F}$-theory SU(5) models is a generic feature without Proton Decay.

* RPV couplings in local $\mathcal{F}$-theory SU(5) models can be study in a $SO(12)$ point of enhancement.

* At the GUT scale may be naturally suppressed over large regions of the parameter space.

* LLe^c and $u^c d^c d^c$ (especially with the heaviest generations) type of RPV interactions from $\mathcal{F}$-theory are expected to be within current bounds.

* Study of other cases where generations resides in the same matter curve or mixed (2+1) cases.

Thank you !

back up

⋆ For the bottom and tau coupling we have:

$$y_{b,\tau} = m_*^4 \; t_{abc} \int_S \mathrm{det}(\vec{\psi}^{(b)hol}_{10_M}, \vec{\psi}^{(a)hol}_{5_M}, \vec{\psi}^{(c)hol}_{5_H}) \mathrm{dVol}_S$$

$$= m_*^4 \; t_{abc} \; \mathrm{det}(\vec{v}^{(b)}, \vec{v}^{(a)}, \vec{v}^{(c)}) \int_S \chi^{(b)hol}_{10_M} \chi^{(a)hol}_{5_M} \chi^{(c)hol}_{5_H} \mathrm{dVol}_S.$$

⋆⋆ A similar formula can be written down for the RPV coupling:

$$y_{RPV} = m_*^4 \; t_{abc} \int_S \mathrm{det}(\vec{\psi}^{(b)hol}_{10_M}, \vec{\psi}^{(a)hol}_{5_M}, \vec{\psi}^{(c)hol}_{5'_M}) \mathrm{dVol}_S$$

$$= m_*^4 \; t_{abc} \; \mathrm{det}(\vec{v}^{(b)}, \vec{v}^{(a)}, \vec{v}^{(c)}) \int_S \chi^{(b)hol}_{10_M} \chi^{(a)hol}_{5_M} \chi^{(c)hol}_{5_M} \mathrm{dVol}_S.$$

⋆⋆⋆ t_{abc} is a group factor. Computing the Integral we have

flux constraints

The presence of a chiral state in a sector with root ρ is given if $\det m_\rho > 0$. Depending on the sign of N_Y, the above conditions define different regions of the flux density parameter space.

$-$	$M < \frac{\tilde{N}_Y}{3}$	$\frac{\tilde{N}_Y}{3} < M < \frac{-\tilde{N}_Y}{6}$	$\frac{-\tilde{N}_Y}{6} < M < -\tilde{N}_Y$	$-\tilde{N}_Y < M$
$(N_a - N_b) < \frac{-N_Y}{2}$	None	None	None	None
$\frac{-N_Y}{2} < (N_a - N_b) < \frac{N_Y}{3}$	None	None	QLd^c	QLd^c, LLe^c
$\frac{N_Y}{3} < (N_a - N_b)$	None	$u^c d^c d^c$	QLd^c, $u^c d^c d^c$	All

Table : Regions of the parameter space and the respective RPV operators supported for $\tilde{N}_Y \leq 0$, $N_Y > 0$