Generalization of the Starobinsky model in f(R) supergravity

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Dedicated to Peggy Kouroumalou, colleague and friend

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Motivation

- Long history of attempts to generalize GR considering higher order curvature terms
- An example from cosmology is Starobinsky’s model of inflation and the associated $R + R^2$ theory
- $N=1$ supergravity has also been extensively studied, in an effort to encompass a unified picture of particle theory, including gravity
Chiral Lagrangians

- **N = 1 SUGRA Lagrangian:**

\[
\mathcal{L} = \int d^4 \Theta E^{-1} \Omega(S, \bar{S}) + \left( \int d^2 \Theta 2\mathcal{E} W(S) + h.c. \right)
\]

where \( S \) denotes all chiral multiplets coupled to gravity.

- This can be cast in chiral form:

\[
\mathcal{L} = \int d^2 \Theta 2\mathcal{E} \left( -\frac{1}{8} (\overline{D} - 8\mathcal{R}) \Omega(S, \bar{S}) + W(S) \right) + h.c.
\]

This is particularly useful since any N = 1 supergravity action can be written as a chiral action which includes chiral multiplets and their corresponding kinetic multiplets.
The gravity sector is described by the chiral superspace density $\mathcal{E}$, the supervierbein determinant $E$ and the chiral superspace curvature $\mathcal{R}$. Ignoring fermionic components:

$$\mathcal{E} = \frac{e}{2} \left(1 - \Theta^2 \overline{M}\right)$$

$$\mathcal{R} = -\frac{M}{6} + \Theta^2 \left(\frac{R}{12} - \frac{M\overline{M}}{9} - \frac{b_\mu^2}{18} + \frac{i}{6} D_\mu b_\mu\right)$$

A Poincaré chiral multiplet is given by:

$$\Phi = A + \Theta^2 F$$
Given a chiral multiplet $\Phi$, another chiral multiplet can be constructed whose scalar component includes $F$. This is called the kinetic multiplet $T(\Phi)$ and is equal to:

$$T(\Phi) = \left( \bar{F} - \frac{M}{3} \bar{A} \right) + \Theta \Theta \left[ \Box \bar{A} + \ldots \right]$$

Kinetic multiplets of chiral fields can also be expressed as the projection of their anti-chiral counter part $\Phi$:

$$-\frac{1}{4} \left( \bar{D}^2 - 8 \bar{R} \right) \bar{\Phi} = T(\Phi)$$

For the case of $\mathcal{R}$, keeping only $R$ related terms:

$$T(\mathcal{R}) = \frac{1}{12} R + \ldots + \Theta \Theta \left[ \frac{1}{36} \bar{M} \mathcal{R} + \ldots \right]$$

We can follow the same recipe and get the successive kinetic multiplets, $T(T(\mathcal{R}))$, $T(T(T(\mathcal{R})))$. 
We can then build actions involving the chiral multiplets $\mathcal{R}, T(\mathcal{R}), T(T(\mathcal{R}))$ and so on, as well other chiral multiplets, which we denote collectively by $X$. The general form of those Lagrangians is:

$$\mathcal{L} = \int d^4\Theta E^{-1} \Omega_0(X, \overline{X}, \mathcal{R}, \overline{\mathcal{R}}, T(\mathcal{R}), T(\overline{\mathcal{R}}), ...) +$$

$$+ \left( \int d^2\Theta 2\xi W_0(X, \overline{X}, \mathcal{R}, \overline{\mathcal{R}}, T(\mathcal{R}), T(\overline{\mathcal{R}}), ...) + h.c \right)$$

Such Lagrangians describe higher $R$ theories, by construction, but these are equivalent to standard $N = 1$ supergravities, in which only the Einstein term appears (Cecotti).
By introducing Lagrange multipliers, $\Lambda, \Lambda_1, \Lambda_2...$, one can show (Cremmer et al.) that this is equivalent to a standard $N = 1$ supergravity described by the following functions:

\[
\Omega = \Omega_0(X, \overline{X}, J_1, \overline{J_1}, J_2, \overline{J_2}...) - (\Lambda + \overline{\Lambda}) - 2 (\Lambda_1 \overline{J_1} + J_1 \Lambda_1) - ... - 2 (\Lambda_n \overline{J_n} + J_n \Lambda_n)
\]

\[
W = W_0(X, J_1, J_2, ...) + 2\Lambda J_1 + 2\Lambda_1 J_2 + ... + 2\Lambda_n J_{n+1}
\]

In these neither $\Omega_0$ nor $W_0$ depend on $\Lambda, \Lambda_1, \Lambda_2...$. The theory described by $\Omega, W$ is then written as

\[
\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\Lambda
\]
Lagrange multipliers

The part of the Lagrangian dependent on the Lagrange multipliers is given by:

\[ \mathcal{L}_{\Lambda} = 2 \int d^2 \Theta 2 \epsilon \left[ \Lambda (J_1 - \mathcal{R}) + \ldots + \Lambda_n (J_{n+1} - \mathcal{T}(J_n)) \right] + h.c. \]

In deriving this, we have used the important relation:

\[ \int d^4 E^{-1} (S \overline{H} + \overline{S} H) = \int d^2 \Theta 2 \epsilon S \mathcal{T}(H) + h.c. \]

Plugging the solutions \( J_1, J_2, \ldots, J_n \) into \( \mathcal{L}_{\Lambda} \), it vanishes, so this proves the equivalence of the two theories.
As an instructive, well known example, consider the no-scale supergravity model described by:

$$\Omega = -3 \left( T + \bar{T} - \Phi \bar{\Phi} \right), \quad W = 3\mu \Phi \left( T - \frac{1}{2} \right)$$

where we have one Lagrange multiplier $\Lambda = 3T$ and $J_1 = \mu \frac{\Phi}{2}$. The chiral Lagrangian is given by:

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E} \left( \frac{3}{2} \Phi T(\Phi) + W_0(\Phi) + 6T \left( \frac{\mu}{2} \Phi - R \right) \right) + h.c.$$  

Solving $\frac{\delta \mathcal{L}}{\delta T} = 0$ and using the forms of $\mathcal{E}, R, T(R)$ we indeed get

$$e^{-1}\mathcal{L} = -\frac{R}{2} + \frac{R^2}{12\mu^2} + \ldots$$
The Starobinsky example

The bosonic part of N=1 supergravity Lagrangian, along the direction $\Phi = 0$ has the form:

$$e^{-1}\mathcal{L} = -\frac{R}{2} - 3\frac{\left|\nabla_\mu T\right|^2}{(T + \bar{T})^2} - 3\mu^2\frac{\left|T - \frac{1}{2}\right|^2}{(T + \bar{T})^2}$$

which, by freezing $\text{Im}T$ to zero and by redefining $\text{Re}T = \frac{1}{2} e^{\frac{2}{3}\phi}$

$$e^{-1}\mathcal{L} = -\frac{R}{2} - \frac{1}{2} (\nabla_\mu \phi)^2 - \frac{3\mu^2}{4} \left(1 - e^{-\sqrt{\frac{2}{3}\phi}}\right)^2$$
The strategy outlined in the previous section can be employed to construct higher derivative supergravity theories, by using at least two Lagrange multipliers. Consider:

\[ \Omega = T + \overline{T} + (Q\Phi + \Phi\overline{Q}) + \omega(X, \overline{X}, \Phi, \overline{\Phi}, C, \overline{C}) \]

\[ W = T\Phi + QC + h(X, \Phi, C) \]

The correspondence with the previous section is:

\[ T = -\Lambda, \Phi = -2J_1, Q = \Lambda_1, C = J_2 \]

Solving for \( T, Q \) we get

\[ \Phi = -2\mathcal{R}, C = -T(\Phi) = 2T(\mathcal{R}) \]
Higher derivative $f(R)$ theories: an example

To demonstrate this, consider the example

$$\omega = 2\alpha C\bar{C}, \, h = 0$$

This leads to

$$\mathcal{L} = 8\alpha \int d^4\Theta E^{-1} \, T(R) \overline{T(R)} = 4\alpha \int d^2\Theta 2\mathcal{E} \, T(R) \, T(T(R)) + h.c.$$ .

If we collect only curvature dependent terms, we get:

$$e^{-1}\mathcal{L} = \frac{\alpha}{18} \left( \frac{R^3}{6} + R\Box R \right) + \ldots$$

Problems: In the ordinary $N=1$ supergravity there are negative eigenvalues of the complex scalar kinetic matrix. Also, we get a mismatch of the d.o.f.
An alternative approach: Q deformations

- $\omega, h$ now are allowed to have Q dependencies. This means that $Q (q, F_q)$ is no longer a Lagrange multiplier and becomes dynamical in the theory.

We take the most generic $\omega$:

$$\omega = \sum \alpha_{n_1 m_1} (X^{n_1} \Phi^{n_2} C^{n_3} Q^{n_4}) (X^{m_1} \Phi^{m_2} C^{m_3} Q^{m_4})$$

The chiral form of the Lagrangian is then:

$$\mathcal{L} = \int d^2 \Theta \epsilon \mathcal{E} P(X, \Phi, Q, C) + h.c$$

where the superpotential function $P$ is:

$$P = h (X, \Phi, Q, C) + T (\Phi + 2R) + Q (C + T(\Phi)) + \sum f(X, \Phi, Q, C) T(g(X, \Phi, Q, C))$$
Q deformations: an example

The equations of motion for any superfield $A$ are given by:

$$\frac{\partial H}{\partial A} + \sum \left( \frac{\partial f}{\partial A} T(g) + \frac{\partial g}{\partial A} T(f) \right) = 0$$

Consider for instance:

$$\omega = 2\alpha C\overline{C} + 2\lambda Q\overline{Q} + 2\beta \Phi\overline{\Phi}$$

$$\frac{\partial}{\partial T} : \Phi = -2\mathcal{R}, \quad \frac{\partial}{\partial Q} : C + T(\Phi) + 2\lambda T(Q) = 0$$

which leads to:

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E} [h(\Phi, C) + 4\alpha T(\mathcal{R}) T(T(\mathcal{R})) + 4\beta \mathcal{R} T(\mathcal{R}) -$$

$$-8\alpha\lambda T(\mathcal{R}) T(T(Q)) + 4\alpha\lambda^2 T(Q) T(T(Q)) - \lambda QT(Q)] + h.c.$$
Models with $h = \Phi f(C)$

The simplest non trivial choice leads to generalizations of the
supersymmetric Starobinsky model, i.e. ”deformed” Starobinsky models.

$$\Omega = -3 \left( T + \overline{T} - \Phi \overline{\Phi} \right) - \mu \left( Q \Phi + \Phi Q \right) + 2\lambda Q \overline{Q} + 2\alpha C \overline{C}$$

$$W = 3\mu \Phi \left( T - \frac{1}{2} \right) + QC + \mu \Phi \left( f(C) - \frac{3}{2} \right)$$

which leads to:

$$e^{-1} \mathcal{L} = -\frac{R}{3} f\left( \frac{R}{6} \right) + \frac{\alpha}{18} \left( \frac{R^3}{6} + R \Box R \right) + \frac{\beta}{18} R^2 + \ldots$$
The mass spectrum of the dual theory

In order to read the mass spectrum, we collect all quadratic terms of the total Lagrangian. For example, for the mixing of the the graviton with $\hat{S}$, the real part of $F_q$, we have:

$$e^{-1} \mathcal{L}_{RS}^{quad} = F(R) + \frac{\alpha}{18} R \Box R - \frac{\sqrt{\alpha}}{3} R \Box \hat{S} + \frac{f_0'}{6 \sqrt{\alpha}} R \hat{S} + \frac{1}{2} \hat{S} \Box \hat{S} - \frac{1}{8 \alpha \lambda} \hat{S}^2$$

The mass spectrum, arising from this mixing, includes two massless states, the graviton in two helicity states, and two massive Klein Gordon fields. For the latter:

$$m_{1,2}^2 = \frac{B \pm (B^2 - 4 \alpha (4 \beta \lambda - 1) f_0) \frac{1}{2}}{2 \alpha (4 \beta \lambda - 1)}$$

where

$$B = \lambda (f_0')^2 - f_0' + 4 \alpha \lambda f_0 + \beta$$

We notice a degeneracy in all masses calculated in this model, as the same masses appear for the $\Psi, G, A, \rho$ and $B, \sigma$ sectors.
Supergravity in the Einstein frame

The previous models are dual descriptions of standard $N = 1$ supergravities where the dependence on the curvature enters with the canonical Einstein form, $-\frac{R}{2}$. In this frame kinetic and potential terms are given by:

$$e^{-1} \mathcal{L}_{\text{kin}} = -K^{-1} J_I \partial^\mu \phi^J \partial_\mu \phi^I$$
$$e^{-1} \mathcal{L}_V = -e^K (F^I F_I - 3 |W|^2)$$

The complete expression for the potential is lengthy but it receives a quite simple form along the specific directions $\Phi = Q = 0$ for which there is a minimum. This takes the form:

$$V = 9 \left( \frac{(4\beta\lambda - 1) |C|^2 + |C - 2\lambda (T + f(C))|^2}{2\lambda (4\beta\lambda - 1) (T + \overline{T} + 2\alpha |C|^2)^2} \right)$$
Examining the potential

The cosmological evolution leads to at least a two-field inflation model. For vanishing values of $C$ we actually have a Starobinsky potential that passes through the minimum.

By redefining $ReT = -f_0 e^{\sqrt{\frac{2}{3}} \psi}$:
Examining the potential

Starting from some initial $\psi, ReC$, the field $ReC$ rolls down to reach a minimum in the $ReC$-direction, that is $\frac{\partial V}{\partial ReC} = 0$ as one expects from the profile of the potential. Then it stays on the trajectory $\frac{\partial V}{\partial ReC} = 0$, with the $\psi, ReC$ fields being decreased, trying to reach the absolute minimum, corresponding to values $\psi = 0, ReC = 0$. The inflationary trajectory, $\frac{\partial V}{\partial ReC} = 0$ tracks the Starobinsky trajectory $ReC = 0$, and approaches it towards the end of the inflationary journey.
Conclusions

- We have seen that the departure from the linearity of the Q-field, that plays the role of a Lagrange multiplier when it linearly appears in the theory, besides leading to a standard supergravity theory free of ghosts it also yields supergravity models which generalize the supersymmetric Starobinsky models in an interesting manner.

- In the presented model the potential arising in the Einstein frame has interesting features. It exhibits Starobinsky direction as in the supersymmetric completion of $R + R^2$ theory with no extra instabilities developed.

- The cosmological evolution seems to lead genuinely to at least two field inflation. The systematic study of the cosmological properties of this model is currently under investigation.
Thank you!