From Linear to Non-Linear SUSY and Back Again

Fotis Farakos
Padova U. & INFN

HEP 2017, Ioannina
Supersymmetry is spontaneously broken.

It is described by non-linear realizations.

This can be done with constrained superfields.
Applications

→ Low energy theories. Brignole, Feruglio, Zwirner ’97, Antoniadis, Dudas, Ghilencea, Tziveloglou ’10, Goodsell, Tziveloglou ’14

→ Effective models for inflation. Antoniadis, Dudas, Ferrara, Sagnotti ’14, Ferrara, Kallosh, Linde ’14, Dall’Agata, Zwirner ’14, Dall’Agata, FF ’15

→ Current dS phase of our universe. Dudas, Ferrara, Kehagias, Sagnotti ’15, Bergshoeff, Freedman, Kallosh, Van Proeyen ’15, Hasegawa, Yamada ’15, Cribiori, Dall’Agata, FF, Porrati ’16
How to describe the SUSY breaking sector?

How to describe matter?

Is the description general?
SUSY breaking sector
F-term breaking

Break SUSY with a chiral superfield \((\overline{D}_\alpha X = 0)\)

\[
X = A + \sqrt{2} \theta^\alpha G_\alpha + \theta^\alpha \theta_\alpha F.
\]

When SUSY is broken:

\(
\langle F \rangle \neq 0.
\)

Goldstino: \(\delta G_\alpha = -f \epsilon_\alpha + \cdots\)

The scalar \(A\) is generically massive.
In the low energy the scalar can be removed from the spectrum by imposing
\[ X^2 = 0 \quad \rightarrow \quad A^2 + 2\sqrt{2}\theta(AG) + 2\theta^2(2AF - G^2) = 0 \]
\[ \rightarrow \quad A = \frac{G^2}{2F}. \]

This is equivalent to decoupling \( A \).

The superfield \( X \) now is
\[ X = \frac{G^2}{2F} + \sqrt{2}\theta^\alpha G_\alpha + \theta^\alpha \theta_\alpha F. \]
The simplest supersymmetric Lagrangian is

\[ \mathcal{L} = \int d^4 \theta \bar{X} X + \left\{ \int d^2 \theta f X + c.c. \right\}. \]

In component form (after eliminating \( F \)) we get

\[ \mathcal{L} = -f^2 + i \partial_m \bar{G} \sigma^m G + \frac{1}{4f^2} \bar{G}^2 \partial^2 G^2 - \frac{1}{16f^6} G^2 \bar{G}^2 \partial^2 G^2 \partial^2 \bar{G}^2. \]

Komargodski, Seiberg '09

Fermion D.O.F. \( \neq \) Boson D.O.F.

\[ \delta G_\alpha = -f \epsilon_\alpha - (i/2f) \sigma^m_\alpha \bar{\epsilon}^{\dot{\alpha}} \partial_m G^2 + \cdots \]
Matter sector
Matter fields reside in superfields.

In the low energy some fields are heavy and decouple.

This is described by constraints on matter superfields.
We have another chiral superfield \( \overline{D}_{\dot{\alpha}} Y = 0 \)

\[
Y = y + \sqrt{2} \theta \chi^y + \theta^2 F^y.
\]

We can have

\[
X Y = 0.
\]

This gives

\[
y = \frac{G \chi^y}{F} - \frac{G^2}{2F^2} F^y.
\]
Keeping only scalars
Komargodski, Seiberg ’09

- We have another chiral superfield \((\overline{D}_\alpha H = 0)\)
  \[
  H = h + \sqrt{2}\theta \chi^H + \theta^2 F^H.
  \]

- We can have
  \[
  \overline{X} D_\alpha H = 0.
  \]

- This gives
  \[
  \psi^H = \sqrt{2}i \sigma^m \left( \overline{\frac{G}{F}} \right) \partial_m h,
  \]
  \[
  F^H = \left( \frac{\overline{G}^2}{2F^2} \partial^2 h - \partial_n \left( \overline{\frac{G}{F}} \right) \overline{\sigma^m \sigma^n} \overline{\frac{G}{F}} \partial_m h \right).
  \]
Removing any selected component

For a generic superfield

\[ Q = q + \theta \chi^q + \cdots \]

We propose the (irreducible) constraint

\[ X \bar{X} Q = 0. \]

This removes only the lowest component

\[ q = \frac{G \chi^q}{\sqrt{2F}} + \cdots \]
From irreducible constraints

→ All known constraints are explained.

→ We can build new constrained superfields.

→ This corresponds to formal decoupling of eliminated fields.
Is this description general?
The mass of the scalar is generated from the Kähler potential term

$$\delta K = -\lambda X^2 \bar{X}^2.$$ 

In the formal limit

$$\lambda \to \infty$$

the sgoldstino gets an infinite mass and decouples.

The superspace equations of motion have a $\lambda$ part which should vanish identically

$$\bar{D}^2(XX^2) = 0 \to X^2 = 0.$$ 

Often questioned if there always exists in the IR $X^2 = 0$. Dudas, Gersdorff, Ghilencea, Lavignac, Parmentier ’11
The goldstino and the Ferrara–Zumino supercurrent

For every chiral model we have \((\text{Ferrara, Zumino '74})\)

\[
\overline{D} \dot{\alpha} J_{\alpha \dot{\alpha}} = D_\alpha Z ,
\]

where

\[
Z = 4W - \frac{1}{3} \overline{D}^2 K .
\]

It has been proposed that in the IR when SUSY is broken, on-shell one finds

\[
Z \sim X \rightarrow Z^2 = 0 . \quad \text{Komargodski, Seiberg '09}
\]
Counterexample: The $X–Y$ system in the IR

- We can study for example the system

$$X^2 = 0, \quad X Y = 0.$$  

- For

$$\mathcal{L} = \int d^4\theta \left( X\bar{X} + Y\bar{Y} \right) + \left\{ \int d^2\theta \left( f X + g Y \right) + c.c. \right\},$$

we find

$$Z^2 = \frac{64}{9} g^2 Y^2 \neq 0.$$  

But is there always the $X^2 = 0$ constrained superfield?
Break SUSY with a chiral superfield $\Phi$ and parametrize it (independent of UV details) as

$$\Phi = X + S,$$

using the constrained superfields

- Goldstino: $X^2 = 0$.
- Sgoldstino: $\bar{X} D_\alpha S = 0$.

Equivalently in components

$$A^\Phi = s + \frac{G^2}{2F},$$
$$\chi^\Phi = G + \mathcal{O}(G),$$
$$F^\Phi = F + \mathcal{O}(G).$$
Example: Decoupling the Sgoldstino

Consider

\[ \mathcal{L} = \int d^4 \theta \left( \Phi \bar{\Phi} - \mu \Phi^2 \bar{\Phi}^2 \right) + \left\{ \int d^2 \theta f \Phi + \text{c.c.} \right\} \]

\[ = \int d^4 \theta \left( |X|^2 + |S|^2 - \mu \left[ 4|X|^2|S|^2 + |S|^4 \right] \right) \]

\[ + f \left( \int d^2 \theta (X + S) + \text{c.c.} \right). \]

In the IR the scalar decouples and we find

\[ S = 0, \]

and (1) becomes the V–A.

For more complicated models, the decoupling of S might be more complicated, but this cannot change the \( X^2 = 0. \)
Conclusions
→ Always $X^2 = 0$ in the IR for F-term and D-term breaking.
   *Cribiori, Dall’Agata, FF ’17*

→ $X$ not essentially aligned with $\mathcal{Z}$.

→ Use the constraint $X\overline{X}Q = 0$ for matter.
   *Dall’Agata, Dudas, FF ’16*

→ Supergravity?
Thank you!